Positioning with WLAN

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July 16, 2004

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Abstract

Determining one's current position using only the information received from WLAN base station access points is a challenging task. Several positioning systems with a varying degree of complexity have been proposed. The positioning technique used in this report differs from most of the other techniques in one critical aspect. Our positioning system does not take into account the positions of the WLAN base stations that the device receives signals from, it just registers an identifier for those base stations, e.g. their MAC addresses, and the signal strengths received. Therefore, the goal of such a scheme cannot be to deduce coordinates indicating the current position, instead the logical position, e.g. a room, floor or area, represented by a name (such as "Building IFW - Room C21") is to be determined.
Another aspect, in which our system is different, is the statistical approach,

which makes it possible to specify a confidence interval, yielding a simple procedure to decide whether certain signals are compatible with a specific logical position or not.

1 Introduction

In Section 1.1 the problem is described and in Section 1.2 a selection of other proposed systems is introduced. Section 2 presents the system model and provides insight into the statistical approach taken therein. While Section 3 presents a possible implementation, Section 4 contains a collection of experimental results obtained using the aforementioned implementation. Finally, in Section 5 the system model is analyzed, based on the experimental results.

1.1 Problem Description

Positioning using only identifiers of base stations and the received signal strengths is hard [1]. Due to noise and interference, the signal strengths vary remarkably. Scattering and reflection of electromagnetic waves can strongly distort the signals [2], whereas shielding severely attenuates the signal strength. A single person can alter the signal strength by up to -3.5dBm [3]. What is more, when measuring the signal strengths, a high level of measurement noise is to be expected, especially when using standard hardware [4]. Another potential problem is that base stations can fail or they can simply be turned off.

Considering those problems, the question arises to what extent or whether it is feasible at all to determine a user's logical location. A system ought to be robust in the sense that, even if some base stations fail or are shielded temporarily, the outcome of the system should not deteriorate quickly. Clearly, the system also has to take into account the potentially high variance of the signal strengths. Fortunately, there is a strong correlation between the distance of a base station and the signal strength [5], which can certainly be exploited in the system. If the problem of the high variance can be tackled, then finding the current logical position with a high probability appears to be feasible.

1.2 Related Work

The goal of the majority of the systems presented in this section is to determine the mobile user's physical position. There is one system, called Nibble [6], that is similar to our system in that it also works with logical locations, yet the system models differ. Nibble's modular probabilistic approach for inferring location uses Bayesian networks. A Bayesian network is a graphical representation of a joint probability distribution that explicitly declares dependency relationships between random variables in the distribution.

In the following systems, the physical position of the mobile user is approximated. Therefore, the geometry of the surrounding area has to be known to a certain extent. Even the knowledge of the exact positions of all base stations is often a prerequisite. Due to the correlation between signal strength and distance, one simple approach used in the RADAR system [3] to solve the positioning problem is to build up a so-called radio map, which stores position-signal strength pairs. In order to approximate the user's current position, the average of the k nearest neighbors is returned. A problem of this approach is the choice of k. A variation of this approach is the joint clustering technique [7]. A cluster is a set of positions where signals from exactly the same base stations are received. This set of base stations is denoted the cluster key. Once the right cluster is found in the online phase, Baye's theorem is used to determine the probability of each location within the cluster. The critical aspect here is the choice of the dimension of the joint distribution. An advantage is surely the use of a probability distribution for the signal strengths. It has been stated that an optimal strategy must consider the probability distribution of the signal strengths and that taking the average of several signals received reduces the error [8].

Another system, called GPPS [4], uses a maximum likelihood estimator. Gaussian process models are built for the distribution of the signal strengths, using the Matérn kernel function. The resulting maximum likelihood estimator is returned as the solution, i.e. there is no error bound. The positions of all the base stations ought to be known, otherwise they are approximated.

A totally different approach is used in the LEASE system [9]. This infrastructure based system uses a small number of stationary emitters and sniffers in order to locate the user. The sniffers collect information about the user and the stationary emitters. Afterwards, the collected information is forwarded to the location estimation engine (LEE), which knows the positions of the stationary emitters. A similar system called Palantir has been proposed [10]. In this system, the floor of a building is divided into grids and the signal strength in the middle of each grid is approximated again using sniffers, which are the main component of this system. A nearest neighbor search is performed in order to approximate the user's current physical position.

Another interesting idea is the combination of various localization techniques [11]. Since it's not clear, whether a single positioning algorithm can find the optimal solution, several different methods are combined in order to achieve more accurate results. The contributions of all methods have to be weighted to minimize erroneous information.

2 System Model

In our system model, the positions of the base stations are not required. In the so-called calibration phase, the system learns the logical mapping between base station identifers, their signal strengths and the logical locations. The system memorizes this mapping and uses it in the online phase, when the current logical position is to be determined.

The system allows for the specification of a confidence interval. The result of the system is the logical location that best matches the received signals. However, if the measured signals are not compatible with any logical location, considering the given confidence interval, then no solution is to be returned. Alternatively, all logical locations that are compatible with the given confidence interval can be returned.

2.1 Calibration Phase

Let M denote the mobile user and b_i denote the *ith* base station. b_i is a pair (a_i, s_i), where a_i is the identifier of the base station and s_i is the received signal strength, in dBm or mW. M collects data in order to characterize a logical location L. For that purpose, M performs k measurements within that logical location.

A measurement is a set of pairs b_i=(a_i, s_i), i.e. a measurement has the form $S^{(j)}$=$\{b_1, b_2, \dots b_n\}$, where j $\in$ [1,k]. n is the number of different base stations from which signals are received. Technically, n is not known until the *kth* measurement has been carried out. Each measurement probably receives a signal from one or more base stations that another measurement does not and vice versa. If there is no signal of b_i in the *jth* measurement, then $s_i^{(j)}$ is set to 0.

The characteristics of L have to be determined utilizing this data. First, $\overline{S}$ is calculated, which for each a_i stores all the average signal strengths $\overline{s}_i$, i.e. $\overline{S}$= $\{(a_1, \overline{s}_1), \dots, (a_n, \overline{s}_n)\}$, where $\overline{s}_i = \frac{1}{k}\sum_{j=1}^{k} s_i^{(j)}\ \forall\ i \in [1, n]$. Either all b_i are considered, or the dimension is reduced by excluding those base stations from which signals were rarely received in those k measurements.[1] Assuming that such a reduction has been performed, let there be m $\leq$ n base stations that did not violate this constraint. Because of noise and interference, it is surely not sufficient to consider only $\overline{S}$ as the characteristics for L. In order to take those factors into account, a probabilistic approach seems to be appropriate. The normal distribution is used as an approximation of the distribution of the signal strength. For that purpose $\overline{S}$ has to be extended to include the variance v_i of each of its associated base stations, i.e. $S^* = \{(a_1, \overline{s}_1, v_1), \dots, (a_m, \overline{s}_m, v_m)\}$ and $v_i = \frac{1}{k-1}\sum_{j=1}^{k}(s_i^{(j)} - \overline{s}_i)^2\ \forall i \in [1, m]$ where S^* is the extended form of $\overline{S}$. Not only does S^* describe the average signal strengths, but also the variance of the individual contributions, therefore S^* captures the characteristics of a particular location more precisely.

2.2 Online Phase

In the online phase, the system is interrogated. In order to derive M's current logical location, a fresh set of measurements $S = \{b_1, b_2, \dots, b_d\}$ is acquired. Let $\mathscr{L} = \{L_1, L_2, \dots, L_p\}$ be the set of all logical locations known to the system

[1]This reduction is included in the implementation presented in Section 3. A parameter describes how often a base station has to appear, given the number of measurements k.

and let n_i be the dimension of L_i, i.e. the number of base stations constituting the characteristics of L_i. In addition, let S_i^* denote the characteristic set of L_i.

Given that all measurements are probabilistic, it is desirable to find the logical location $L^* \in \mathscr{L}$ that maximizes the probability of M being at this location, as opposed to being at any of the other locations of $\mathscr{L}$. Naturally, L^* also has to respect the given confidence interval. As mentioned in Section 2, alternatively all locations that comply with the confidence interval could be given as the result.

In order to achieve these goals, the value Z_i is calculated for each logical location of $\mathscr{L}$ according to the following formula.

$$Z_i := \sum_{j=1}^{n_i} \frac{(\overline{s}_{ij} - s_j)^2}{v_{ij}} \tag{1}$$

$\overline{s}_{ij}$ and v_{ij} denote the average signal strength of the *jth* base station of the logical location L_i and the variance thereof, respectively. s_j denotes the signal strength obtained from the same base station in the new measurements, that means, the canonical order of S is adapted to S_i^*. If there is a particular base station b_j that appears in S_i^*, but not in S, then s_j is simply set to 0.[2] If on the contrary, there is a base station appearing in S, but not in S_i^*, then it is ignored, since it is not part of the characteristics of L_i. Alternatively, the characteristic signal strength of this base station could be set to 0; however, filtering certain base stations in the calibration phase could not be applied in that case.

The individual terms of Z_i are squared normal variables, hence Z_i is χ^2-distributed with n_i degrees of freedom. L_i can only be considered M's current location, given a confidence interval p, if Z_i is lower or equal to γ_i, where γ_i denotes the value for which it holds that the integral of $\chi^2_{n_i}(t)$ from 0 to γ_i is equal to p, where $\chi^2_{n_i}(t)$ is the χ^2-probability density function with n_i degrees of freedom. In order to obtain the value γ_i, the inverse cumulative density function is used.

$$inverseCumulativeDensity(n_i, p) = \gamma_i \iff \int_0^{\gamma_i} \chi^2_{n_i}(t)\ \mathrm{d}t = p \tag{2}$$

For example, if $n_i = 5$ and $p = 0.95$, then $Z_i \leq 11.07$, otherwise the confidence interval constraint is violated.

Given the Z_i for all logical locations, it is quite simple to determine the best logical position. By choosing the $L_i \in \mathscr{L}$ for which Z_i is minimal, we obtain the solution that both minimizes the least squares error weighted with the inverse of the variance and constitutes the maximum likelihood estimator:
It is clear that the chosen L_i minimizes the least squares error, but it remains to be shown that it is at the same time the maximum likelihood estimator: For that purpose, the characteristic set S^* of one logical location is considered. Since there is a canonical order imposed on the elements of S^*, it is possible to omit the identifiers in each element. Furthermore, all the average signal

[2] This is only reasonable if signal strengths are stored in mW and not in dBm.

strengths and the variances thereof are renamed, in order to emphasize their probabilistic meaning, i.e. S^* has the form $\{(\mu_1, \sigma_1^2), \ldots, (\mu_n, \sigma_n^2)\}$. In order to simplify the formulae, the variances, which are also given, are not displayed. According to our assumption about the distribution of the signal strength, the following formula holds:

$$\forall\ b_i : \quad prob(x|\mu_i) = \frac{1}{\sqrt{2\pi}\sigma_i}\ e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}}. \tag{3}$$

The base stations b_i are the base stations that constitute the characteristics of the logical location with the characteristic set S^*. The formula can be extended to include all those base stations together:

$$prob((x_1, \ldots, x_n)|(\mu_1, \ldots, \mu_n)) = \prod_{i=1}^{n} \Big(\frac{1}{\sqrt{2\pi}\sigma_i}\ e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}} \Big). \tag{4}$$

The definition of $lik((\mu_1, \ldots, \mu_n))$ is useful in the analysis:

$$lik((\mu_1, \ldots, \mu_n)) = prob((x_1, \ldots, x_n)|(\mu_1, \ldots, \mu_n)). \tag{5}$$

Clearly, the maximum likelihood estimator maximizes $lik((\mu_1, \ldots, \mu_n))$. The following formulae show both the function to be maximized and a transformation thereof:

$$\begin{aligned} max\{lik((\mu_1, \ldots, \mu_n))\} &= \prod_{i=1}^{n} \Big(\frac{1}{\sqrt{2\pi}\sigma_i} \Big)\ max\Big\{ \prod_{i=1}^{n} e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}} \Big\} \\ &= \prod_{i=1}^{n} \Big(\frac{1}{\sqrt{2\pi}\sigma_i} \Big)\ max\Bigg\{ e^{\left(-\sum_{i=1}^{n} \frac{(x-\mu_i)^2}{2\sigma_i^2}\right)} \Bigg\}. \end{aligned} \tag{6}$$

It holds that

$$e^{\left(-\sum_{i=1}^{n} \frac{(x-\mu_i)^2}{2\sigma_i^2}\right)} \tag{7}$$

is maximal if and only if

$$\sum_{i=1}^{n} \frac{(x_i - \mu_i)^2}{\sigma_i^2} \tag{8}$$

is minimal. That means, the maximum likelihood estimator is in fact obtained by minimizing the weighted error in the least squares sense.[3]

Finding all logical locations that comply with the given confidence interval p is straightforward: All logical locations L_i for which $Z_i \leq \gamma_i$ holds are possible solutions.

[3]This is true if the distribution of the signal strength is the normal distribution.

3 Implementation

The system, which has been implemented in Java, mainly consists of two parts. One is the Environment class and the other is the SignalRetriever interface. While the Environment class provides all the functionality needed to both calibrate the system and use it in the online phase, the SignalRetriever interface deals with the acquisition of measurements.

3.1 Environment Class

The Environment class offers methods to add, remove and also reset logical locations, which are objects of the Location class. Measurements can be added to any location at any time. For this purpose, the Environment class provides the following method.

- void addMeasurementsToLocation(Measurement[] meas, java.lang.String locName)

Each Location object stores all the BaseStation objects that characterize this location, i.e. the BaseStation objects store the average signal strength and the variance of the signal strengths received from a specific base station. It has to be noted that, in order to be able to update the variance once new measurements are added, it is necessary not only to store the average and the variance, but also all single measurements received. In this implementation, up to 200 measurements are stored. Once new measurements are added and the maximum of 200 is reached, then the oldest measurements are removed. In our implementation, not more than 5 base stations can constitute the characteristics of a certain location. If a new base station appears, then the base station with the least amount of associated measurements is replaced with the new base station. Of course, other strategies are possible to handle this problem.

There are many parameters that can be set in the Environment class, such as the number of measurements to be performed in the calibration and the online phase, the confidence interval and the measurement quotient, which indicates how often signals from an individual base station have to be received, given the number of measurements performed. The Environment class and all associated classes, i.e. the Location and BaseStation class implement the Serializable interface, providing a simple way to store entire environments.

So far, solely the calibration phase has been discussed. The Environment provides two methods for the online phase.

- java.lang.String findBestLocation(Measurement[] meas)
- java.lang.String[] findPossibleLocations(Measurement[] meas)

As the name indicates, the first method is used to find the logical location with the minimal Z_i among all locations. The return value is the name of this location. The second method returns the name of all locations compatible with the confidence interval. They operate exactly as described in Section 2.2.

In particular, the Location class provides two methods which are used in the implementation of the aforementioned methods.

- double getQuadError(Measurement[] meas)
- boolean isPossibleForMeas(Measurement[] meas, float prob)

While the first method calculates the weighted quadratic error, the second method decides, whether the given array of Measurement objects is compatible with this location's characteristics, given the confidence interval *prob*.

3.2 SignalRetriever Interface

Measurements have to be retrieved from the WLAN card. The SignalRetriever interface defines only one method that can be used for this purpose.

- Measurement[] getMeasurements(int n)

The parameter n indicates how many times measurements are retrieved. It is important to note that each Measurement object collects all the signal strengths received from one particular base station, where its MAC address is used as its identifier. The Measurement class could have been defined in such a way that it collects all base stations from which signals were received in one measurement. However, this would complicate the addition of measurements to base stations and therefore the other approach has been chosen.

In our implementation, the class WRAPISignalRetriever, which implements this interface, is used to retrieve the signals. When it is instantiated, it loads the Java Native Interface library WRAPIJNILibrary.dll, which itself uses the WRAPI.dll [12] to retrieve the MAC addresses and the signal strengths of all base stations. Since measurements are performed very quickly, which results in many equal measurements, it is appropriate to delay consecutive measurements, e.g. by using the static method Thread.sleep(long millis) with a suitable parameter.

4 Experimental Results

The experiments can be grouped in two parts. In the first part, which is treated in Section 4.1, the problems and limitations of positioning with WLAN are depicted. The second part focuses on the performance of the system presented in this report. In particular, the effect of parameters, such as the number of measurements performed and the confidence interval are analyzed. This analysis can be found in Section 4.2.

4.1 Problems and Limitations

A critical aspect that ought to be addressed first is whether it is sensible to assume a normal distribution for the signal strengths. Two base stations, one with predominantly strong signals and one with weaker signals have been selected and their signal strengths have been measured. The results are shown in Figure 1 and Figure 2, respectively.

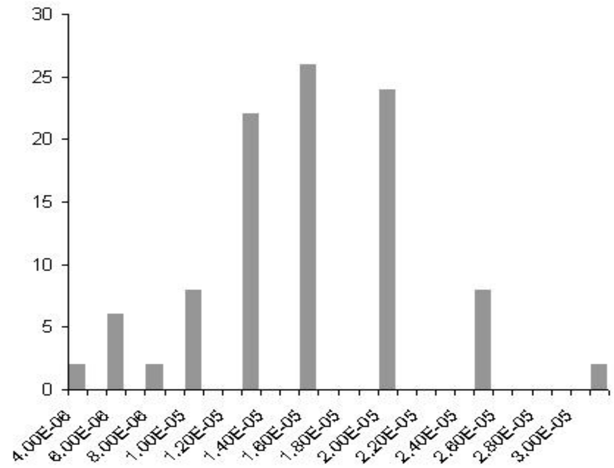

Figure 1: Histogram of measured signal strengths of a base station with strong signals

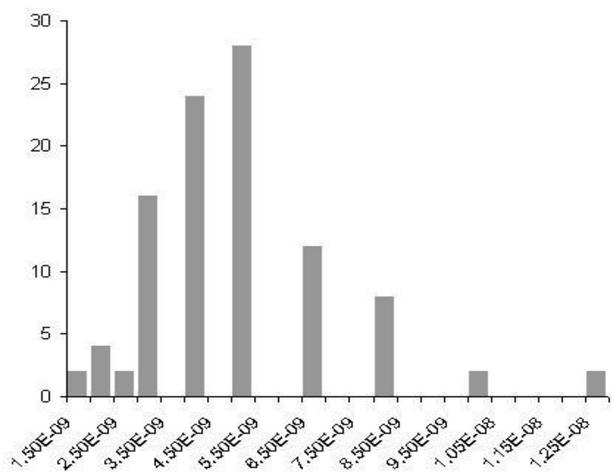

Figure 2: Histogram of measured signal strengths of a base station with weak signals

Even though only 50 measurements have been performed, the histograms show the shape of a normal distribution, thus it can be used as an approximation.

Another aspect that has to be taken into account is the variance of the signal strengths over time. To illustrate the problem, the characteristics of one specific logical location are depicted:

Base station	**Average**	**Variance**
00-07-50-D6-00-FC	1.640386e-7	1.664545e-15
00-07-50-D6-01-0F	2.299025e-5	8.428953e-11
00-07-50-D6-02-F8	2.384267e-9	3.631800e-19
00-07-50-D6-02-FA	2.760481e-6	1.499120e-12
00-07-50-D6-03-CB	4.656942e-9	1.664351e-18

If the characteristics are determined merely 15 minutes later at exactly the same position, the difference is apparent:

Base station	**Average**	**Variance**
00-07-50-D6-00-FC	1.762277e-7	4.949886e-15
00-07-50-D6-01-0F	2.219300e-5	1.057022e-10
00-07-50-D6-02-F8	3.073024e-9	3.603822e-18
00-07-50-D6-02-FA	1.809146e-6	7.098445e-13
00-07-50-D6-03-CB	4.461042e-9	3.218017e-18

While the average signal strength differs from the older measurements by a factor of at most 1.5, the variance reaches a factor of 10. The situation is even worse, when the period of time between those measurements is increased to one day:

Base station	**Average**	**Variance**
00-07-50-D6-00-FC	1.481586e-7	8.501779e-16
00-07-50-D6-01-0F	1.974024e-5	1.851989e-11
00-07-50-D6-02-F8	1.798881e-8	7.769301e-17
00-07-50-D6-02-FA	6.498956e-7	5.176300e-14
00-07-50-D6-03-CB	2.202195e-8	1.167230e-16

In this case, even the average signal strength differs by a factor of up to 7.5, while the variance reaches a factor of 214. It has to be noted that there are differences between base stations. The signal strength of some base stations appear to be more stable than those of others. For example, the base station with the MAC address 00-07-50-D6-02-F8 is responsible for the highest factors as far the average and the variance is concerned.

Not only is there such a high variance over time, the signals measured at different positions in the same room can also vary strongly. In the same room, the following characteristics were obtained at a different position a few seconds after the first measurements:

Base station	Average	Variance
00-07-50-D6-00-FC	8.590684e-9	2.350362e-17
00-07-50-D6-01-0F	9.444324e-6	5.584365e-12
00-07-50-D6-02-F8	2.700181e-9	1.164344e-18
00-07-50-D6-02-FA	1.877516e-5	1.999351e-10
00-07-50-D6-03-CB	2.871098e-7	1.592052e-14

Naturally, the signal strengths vary, but given such a strong deviation, it is scarcely reasonable to consider those measurements the characteristics for the same logical location. This is certainly a limitation of the model presented in this report.

4.2 Accuracy of the System

The question is, given the problems and limitations mentioned, how accurately the system is able to determine the correct position. In one building, the system learnt five logical locations, which in fact are five different rooms. Four of them are on the same floor, while the fifth room (H02) is on the next higher floor.
60 measurements have been performed in the calibration phase with a timeout

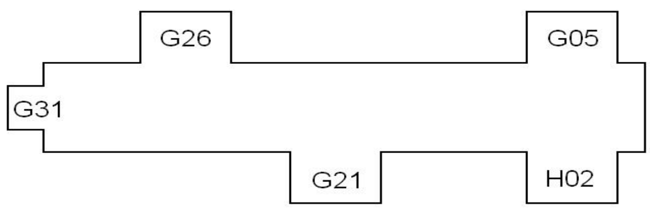

Figure 3: Topology of the building

of 100ms between consecutive measurements. In the first experiment, 10 measurements with the same timeout have been performed in the online phase. The confidence interval has been set to 80%. Since the system has never displayed an incorrect location, i.e. it either displayed the correct answer or it concluded that no location matched the new measurements, it suffices to depict how often it found the right solution:

Room	Match	No match
G05	70%	30%
G21	90%	10%
G26	70%	30%
G31	60%	40%
H02	80%	20%
Total	74%	26%

The accuracy can be improved by prolonging the online phase. In the second experiment, 20 measurements were performed in the online phase with a timeout of 1s between successive measurements:

Room	**Match**	**No match**
G05	75%	25%
G21	50%	50%
G26	75%	25%
G31	100%	0%
H02	100%	0%
Total	80%	20%

Since the system has never returned a bad result, the confidence interval can be increased in order to allow more inaccurate measurements to be considered and thus reduce the frequency of no matches. However, in doing so, the set of possible locations increases and the system starts yielding erroneous results. There is clearly a tradeoff between obtaining a result—which is a certain logical location—in most cases and allowing the system to make mistakes. Therefore, the appropriate setting of the confidence interval is crucial. As an extreme example, in the following experiment, the confidence interval has been set to 100%, i.e. every logical location is considered to be a possible location. On this account, it is only reasonably to retrieve the best matching location. The system then always returns the location with the lowest least squares error:

Room	**Match**	**No match**
G05	90%	10%
G21	90%	10%
G26	100%	0%
G31	100%	0%
H02	100%	0%
Total	96%	4%

Surprisingly, the best matching location is in most cases the right location, indicating that the confidence interval can be set to a high value without rendering the localization scheme inoperative.

5 Conclusion

In Section 4.1, it is shown that it is plausible to assume that the signal strengths are normally distributed. Furthermore, it is shown that it is difficult to determine the characteristics of a logical location, due to the variance of signal strengths over time and the fact that the received signal strengths at different positions of the same logical location vary strongly.

However, the scheme presented in this report is still able to determine the current position of the mobile user with a high probability. In order to achieve a high accuracy, many measurements ought to be performed in the calibration phase. By repeatedly adding measurements after a longer period of time, the problem of the variance over time can be controlled to a certain extent. As illustrated in Section 4.2, the accuracy of the system can be further improved by prolonging the online phase.

With a confidence interval of 80%, the system returned either the right solution or no solution at all. If the confidence interval is set to a higher value, then more positive results are returned, since the logical location with the lowest Z_i is often the right solution. However, by accepting more inaccurate data, at a certain point the system starts making mistakes and returning wrong answers. This indicates that there is a tradeoff between high accuracy and error rate.

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