Eidgenössische Technische Hochschule Zürich Swiss Federal Institute of Technology Zurich

# FIFA World Ranking 

Semester Thesis

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## Abstract

The FIFA/Coca-Cola ranking shows an order of the men football national teams. But this ranking is criticized often. We want to create a new better ranking. To calculate the PageRank of teams we construct a graph based on the matches. The edges and the edge weights are defined by the result and the conditions of the matches. Teams are represented by nodes. The prediction is done with two different prediction models. One compares the PageRank of the team, the second compares the winning probabilities calculated with the raise of the PageRank in case of a victory of both teams. The highest correct prediction rate is $61.51 \%$ for all FIFA matches (except friendlies) between 1999 and the 11th June 2014. For the World Cups 2002, 2006 and 2010 we predicted in $59.38 \%$ of the matches the correct winner. The FIFA/Coca-Cola ranking made $53.65 \%$ correct predictions in World Cup matches.

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## Introduction

2014 is the year of World Cup. The WC is probably the best and fairest way to find the strongest team of the World. But how can we find the best national team during the other years and make a ranking of all the nations? The FIFA has a ranking where all international matches from the last four years are considered and points are earned for ties or wins. This ranking is often criticized. If we have a look on the ranking published on the 5th June, we find Spain on place 1 and Netherlands on 15. During the World Cup in Brazil, Netherlands beat Spain with $5-1$. This is a clear result and the match has shown big differences between the strengths of the two teams. Another fact is that high-ranked teams should avoid matches with little importance against low-ranked teams because they can not earn many points in those matches. If the strong team wins such a match, it might even decrease its point average in the FIFA ranking. But this ranking is important. The groups drawing for World Cup are based on the positions in this ranking.

The aim of this thesis is to create a better fairer ranking. The idea is that we use PageRank and adapt it to football matches. PageRank is already used successfully to rank web pages. We want to use our new ranking, the FootballRank, on the FIFA matches. Before testing FootballRank with FIFA matches we train with a light version on league matches. The Bundesliga seasons between 1975 and 2014 are used to adjust the parameters. The matches of Ligue 1, Premier League, Primera Division and Serie A between 2002 and 2014 are considered to see if our parameter adjustment was good.

We present in the first chapter the official FIFA ranking and some other rankings for tournaments. There is a short introduction to PageRank theory as well. In chapter 3 is our new ranking, the FootballRank, explained. The prediction procedure is described in this chapter as well. In the next chapter we present which matches we use and how we obtained them. In chapter 5 we show the results of testing FootballRank and adjusting the parameters with matches of the Bundesliga and FIFA matches until 1998. We show in chapter 6 the prediction results of the other four leagues (Ligue 1, Premier League, Primera Division, Se-
rie A) and FIFA matches after 1998 with the best parameter settings of chapter 5. In a last chapter we make an outlook.

## Related Work

### 2.1 FIFA/Coca-Cola Ranking

The most famous ranking for international football teams is the FIFA/Coca-Cola Ranking for men[1]. It exists since 1993. The computation of the points for the teams has already changed twice. The current ranking is in use since 2006 and considers all matches in the four last years.

A team earns points for a win or a tie. The amount of points a team $a$ can earn in a match against team $b$ is calculated with: $h_{a, b}=q \cdot\left(l \cdot o_{b} \cdot s\right)$. How those variables are defined we will show in the next lines.
The second part of the equation can be determined before the match. The factor $q$ expresses the result. It is 3 for a victory (not in penalty-shootout), 2 for a victory in penalty-shootout, 1 for a tie or a loss in penalty-shootout and 0 for a loss (not in penalty-shootout).
The match type is defined by $l: 1$ for friendly, 2.5 for continental cup qualifier and World Cup qualifier, 3 for continental cup and Confederations Cup and 4 for World Cup.
The factor $o_{b}$ specifies the strength of the opponent. It is calculated with the ranking position $p_{b}$ of the opponent: $o_{b}=\frac{200-p_{b}}{100}$. There are two exceptions when calculating $o_{b}$ : the number one ranked team yields $o_{b}=2.0$ and teams ranked below 149 give a minimal $o_{b}=0.5$ to opposing teams.
The last factor $s$ is the strength of a confederation and regards the origin of the countries. The average of the two confederation strengths fix $c$. The strenghts are: $\mathrm{UEFA}=1.00, \mathrm{CONMEBOL}=1.00, \mathrm{CONCACAF}=0.88, \mathrm{AFC}=0.86$, $\mathrm{CAF}=0.86, \mathrm{OFC}=0.85$.
There is an additional rule: If a team has not played at least 5 matches in one year the earned points in this time are multiplied by a factor 0.8 for 4 matches, 0.6 for 3 matches, 0.4 for 2 matches and 0.2 for 1 match.

A next rule is that the time span of four years is separated in four time spreads $\delta$. Each covers twelve months. The matches in the last twelve months count more than older matches. This is done with a decreasing time factor $1,0.5,0.3$,
0.2 for the four spreads.

The final ranking points $h_{a}$ for a team $a$ are now calculated by dividing the earned points in a time spread by the number of matches in this $\delta$.
$h_{a}=\frac{\sum_{h_{a, i} \in \delta_{1}} h_{a, i}}{\sum_{h_{a, i} \in \delta_{1}} 1}+0.5 \cdot \frac{\sum_{h_{a, i} \in \delta_{2}} h_{a, i}}{\sum_{h_{a, i} \in \delta_{2}} 1}+0.3 \cdot \frac{\sum_{h_{a, i} \in \delta_{3}} h_{a, i}}{\sum_{h_{a, i} \in \delta_{3}} 1}+0.2 \cdot \frac{\sum_{h_{a, i} \in \delta_{4}} h_{a, i}}{\sum_{h_{a, i} \in \delta_{4}} 1}$
The update of the ranking happens usually each month on a previously fixed day.

An important weakness of this ranking is that the teams have at the end a certain amount of points which are won points divided by the number of matches. The conclusion for strong teams is or should be to not play friendlies against weak teams. Strong teams can not increase their point average in such matches, but might actually decrease it even if they win.
Hosts of the next big tournament have big disadvantages in this ranking. They can only play friendlies and can so only earn points with the smallest match type factor 1. They are punished for a few years in the ranking.
In the ranking of the 5 th June, Italy is on place 9 , Switzerland on 6 . Italy and Switzerland discarded both after the group stage on WC 2010 in South Africa. Italy lost the final of the European Championship in 2012. Switzerland did not even qualify for this Cup. How is it justifiable that Switzerland is higher ranked as Italy?
Another curiosity is that the possible won amount points does not only depend on the strength of the opposing team, it does depend on the origin too. The consequence is that a victory against a team on rank 50 gives might less points than a win against team number 60. We see, FIFA uses might "arbitrary" parameters for its ranking.

### 2.2 Various Rankings

Before we started creating our ranking we observed other ranking types. We will present you them here in a short summary.

One is the ELO ranking. It is used since 1970 in chess tournaments to create the pairings of a tournament, to find the overall champion or just to see the progresses of the players over a time span[2]. Today there even an ELO ranking for football national teams[3]. The ranking is calculated with the old ranking points plus the new points from the newest match. The points for one match depends on the match type, the difference between scored and allowed goals (only by victory), the result and the expected result. There is no time factor used because approximately only the last 30 matches affect the ranking. A big
difference to the FIFA ranking is that it includes an expectation value of the result.

Microsoft uses the TrueSkill ranking which is a development of the ELO rating for its multiplayer games[4]. Each player has two values: a skill (or average performance) and a skill uncertainty. They are calculated with a technique called the Bayesian inference for ranking players. The ranking of a player is the skill minus three times the skill uncertainty. After each game the two values of each player are updated. For the update is only the result of the game important: victory, tie or loss. It does not depend on how clear the victory or loss was. Microsoft does not publish the complete calculations because this ranking is patent protected.

Another ranking for football teams is the Soccer Power Index (SPI) of ESPN[5]. It exists for clubs and national teams. SPI rates a match with the importance of the match, the strength of the opposing team, the home advantage and the allowed and scored goals. The importance of a match depends not on the match type but on the players standing on the field. If there are only young players in the line-up, the importance of the match is smaller. So each player has a rating depending on his last international matches and his matches in Europe's top leagues. The ranking uses a time factor. New matches count more than older and after some time, depending on the number of played matches, old data is deleted. This ranking goes a complete different way than the FIFA or ELO. The output of the ranking are two values for each team: an offensive and a defensive value. The offensive value describes how many goals this team is expected to score against an average team. The defensive value is the expected allowed goals against an average team. It is the only of those rankings that respects the home advantage.

### 2.3 PageRank Theory

Probably everybody with access to the Internet has already used it to search something. There are thousands of search results. The user views often only the ten first listed results. Those need to be good. But how does the search engine decides which pages are listed on the top? Google uses PageRank. This ranking was invented in 1998 by Sergey Brin and Larry Page. The general task of PageRank is to bring order to the web. After a search input, PageRank makes a ranking of the pages. The rating depends on the links of the pages[6].

We now calculate the PageRank of the pages. There are $n$ pages identified with $1, \ldots, n$. The $n$ pages and their links can be described in a directed graph. If there exists a link pointing from page $i$ to page $j$, then there is an edge pointing from page $i$ to page $j$ in our graph. The weight of the edge $(i, j)$ equals the


Figure 2.1: Graph with 4 web pages and 7 links
number of links pointing from page $i$ to page $j$. We can describe the directed graph with the adjacency matrix $Q$ with the dimensions $n \times n$ as well. The entry $Q_{i j}$ ( $i$ indicates the row number, $j$ the column number) is the weight of the edge $(i, j)$.

With this directed graph we can already calculate a ranking: $\mathbf{r}^{\prime}=\mathbf{1}^{n} \cdot Q . \mathbf{1}^{n}$ is a row vector filled with ones. $\mathbf{r}^{\prime}$ is a row vector too. The rank value of page $i$ is defined in column $i$ in $\mathbf{r}^{\prime}: \mathbf{r}^{\prime}(j)$. This ranking calculates for all pages the number of incoming links. Therefore the page with the most incoming links has the highest ranking. The best wep page has the highest ranking.

The weakness of the previous ranking is that it does not matter where the links come from. Taking this fact into account we can adjust our graph by changing the edge weights. The weight of an edge $(i, j)$ is equals the number of links pointing from $i$ to $j$ divided by the $\operatorname{outdeg}(i)$. The outdeg $(i)=B(i)$ is the number of outgoing links of page $i$.


Figure 2.2: Scaled graph with 4 web pages and 7 links

Again, we can describe this graph with a matrix $S$ and calculate again the ranking: $\mathbf{r}^{\prime \prime}=\mathbf{1}^{n} \cdot S$.

We can include the importance of a page from which a link is pointing. The consequence is that the ranking of a page $j$ is higher if there is a high-ranked page $i$ pointing to page $j$. The new ranking is then defined with $\mathbf{r}^{\prime \prime \prime}=\mathbf{r}^{\prime \prime \prime} \cdot S$. The calculation of $\mathbf{r}^{\prime \prime \prime}$ happens now with a starting $\mathbf{r}^{\prime \prime \prime}{ }_{0}$ and updating $\mathbf{r}^{\prime \prime \prime}$ always with the solution of $\mathbf{r}^{\prime \prime \prime} \cdot S$ by using the old $\mathbf{r}^{\prime \prime \prime}$.

In a last step we make the graph ergodic. We add a random surfer with the random surfer parameter $d$ to the system. We connect all pages by inserting edges from all pages $i$ to $j$ with the weight $1-d$. Now we can move from any page $i$ to page $j$ using one link. We need the matrix $A$ with dimensions $n \times n$ and filled with ones to describe this new edges. The ranking is now calculated with

$$
\mathbf{r}=\mathbf{r} \cdot\left(d \cdot S+\frac{(1-d)}{n} \cdot A\right)
$$

That $\mathbf{r}$ is exactly the definition of PageRank.
Our graph represents a Markov chain[7]. The weight on an edge $(i, j)$ is the probability to move from page $i$ to $j$. The solution $\mathbf{r}$ shows the stationary distribution of the Markov chain.

## FootballRank

We create now our new football ranking. We call it FootballRank. It is an adaptation from the PageRank to football matches. We consider again a graph. In this graph, nodes are represented by the football teams. Edge $(i, j)$ is created when there was a match between team $i$ and $j$. The weight and direction of the edge depends on the result and the match conditions.

### 3.1 Edge Weights

A match $m$ consists of six elements: the teams $i$ and $j$, their scores $\mathrm{sc}_{i}$ and $\mathrm{sc}_{j}$, the date of the match $c$, the match status $\Gamma$ and $\tau$ which indicates the home team. All matches $m$ are stored as a sequence in $M=\left(\left(i, j, \mathrm{sc}_{i}, \mathrm{sc}_{j}, c, \Gamma, \tau\right), \ldots\right)$. The set $T$ defines all teams. The size of $T$ is $n$.

### 3.1.1 Respecting Home/Away Team

We use $\tau$ in $m=\left(i, j, \mathrm{sc}_{i}, \mathrm{sc}_{j}, c, \Gamma, \tau\right)$ to indicate the home team:

$$
\tau \leftarrow \begin{cases}0 & \text { if no hometeam } \\ i & \text { if } i \text { is hometeam } \\ j & \text { if } j \text { is hometeam }\end{cases}
$$

### 3.1.2 Result

First we set $s_{i}$ regarding the result of the match $m$ of team $i$ against $j$. A victory is rewarded with $v$, a tie with $t$ :

$$
s_{i} \leftarrow \begin{cases}v & \text { if } m\left(\mathrm{sc}_{i}>\mathrm{sc}_{j}\right) \\ t & \text { if } m\left(\mathrm{sc}_{i}=\mathrm{sc}_{j}\right) \\ 0 & \text { else }\end{cases}
$$

### 3.1.3 Time

In a next step we take the date of the match into account. First we define for FIFA matches a time span $\lambda$ in years. Only matches which are in this $\lambda$ are used to create the graph. For league seasons we choose $\lambda$ between one and four years. For international matches the minimum is four years because national teams play only around nine matches per year. To make a ranking it is good to have a lot of matches in the graph. But it is also important to get rid of old matches after some time. Players leave the team and thus, the strength of a team can change.
We have a second factor that decreases the results according to the time. Older matches inside the time span $\lambda$ should count less than newer matches. We split therefore $\lambda$ in four periods $\delta_{1}, \delta_{2}, \delta_{3}$ and $\delta_{4}$ like the FIFA ranking does. Each period lasts the same time span. The first period $\delta_{1}$ covers the newest time span. The time degradation factor is calculated with $\beta$ which we will change during testing. To calculate $\beta$ we use the date of the match $c$. The date determines $y$ which is the index of the time span in which $c$ is.

$$
\beta^{-(y-1)} \text { with } c \in \delta_{y} \text { and } y=1,2,3,4
$$

It would be possible as well to include the time in the ranking with only one time factor. We have now $\lambda$ and $\beta$. But those two parameters make it easier to create the graph.
Another effect is that teams which do not have played matches in $\lambda$ definitively leave the ranking. If we do not have a $\lambda$, it is possible that teams which do not play matches anymore stay in the ranking forever. The weights of outgoing edges of such teams would decrease because of the time degradation. But we scale all outgoing edges of a node on 1 before we calculate the PageRank. Therefore all those very small weights will be still important inside the ranking.

### 3.1.4 Match Type

Inspired by the FIFA ranking we differentiate matches respecting the match type. We define $\gamma$ as an array with four values. The first value in this array gives the match type factor for matches during the World Cup. The second value defines the factor for continental cups and the Confederations Cups. The third stands for World Cup qualifiers and continental cup qualifiers. The last one is for friendlies. The FIFA uses the same mapping.

$$
\Gamma \leftarrow \begin{cases}\gamma(1) & \text { if World Cup match } \\ \gamma(2) & \text { if continental cup or Confederations Cup match } \\ \gamma(3) & \text { if qualifier match } \\ \gamma(4) & \text { if friendly match }\end{cases}
$$

### 3.1.5 Calculation of Edge Weight

With parameters from the previous section we can calculate now the amount of points $h_{i}(m)$ of a team $i$ in a match against team $j$. The date of the match $c$ defines $y$ with the index of the time period $c \in \delta_{y}$ for $y=1,2,3,4$.

$$
h_{i}(m) \leftarrow s_{i} \cdot \beta^{-(y-1)} \cdot \Gamma
$$

The weight of an edge $(i, j)$ is the sum of all points a team $j$ has earned in matches against $i$ :

$$
w(i, j) \leftarrow \sum_{m \in M(c \in \lambda)} h_{j}
$$

The matches $M$ and the edge weights $w$ describe the graph $D(M, w)$.

### 3.1.6 League Matches

For the league matches is the calculation of $h$ simpler.

$$
h_{i}(m) \leftarrow s_{i}
$$

We do not have different match types and we do not use a time degradation to decrease the weights of old results. But we use a time span $\lambda$.

### 3.2 PageRank Calculation

We have three different variants to calculate the calculate the PageRank of the graph. The differences between them is that we change graph $D$ always a little bit.

### 3.2.1 Variant 1 - Normal

This variant Normal has no modifications. First we change our graph into a Markov chain. We scale the edge weight $w(i, j)$ of $D$ with the sum of all outgoing edge weights from node $i$. And we add the random surfer. The new edge weight $e(i, j)$ in our new graph $E$ is:

$$
e(i, j) \leftarrow d \cdot \frac{w(i, j)}{\sum_{k \in T} w(i, k)}+\frac{(1-d)}{n}
$$

We can now calculate the PageRank $\operatorname{pr}_{E}$ of our new graph $E(M, e)$ with the matches $M$ and weights $e$.

### 3.2.2 Variant 2 - Equality

The second variant is called Equality. The idea of this variant is that if team $i$ wins four times against team $j$ and team $j$ wins one time against team $i$, then $w^{\prime}(i, j)$ should be equal $4 \cdot w^{\prime}(j, i)$. This is often not the case after the scaling. Therefore we modify the weights.

First we scale again the outgoing edge weights in graph $D$ which leads us to the new weight $w^{\prime}$.

$$
w^{\prime}(i, j) \leftarrow \frac{w(i, j)}{\sum_{k \in T} w(i, k)}
$$

We compare the relations between the old edge weight $w$ and the new edge weight $w^{\prime}$ if and only if $w^{\prime}(i, j) \neq 0$ and $w^{\prime}(j, i) \neq 0$ for $i, j \in T$ and $i \neq j$ :

```
if \(\frac{w^{\prime}(i, j)}{w(i, j)}>\frac{w^{\prime}(j, i)}{w(j, i)}\) then
    \(w^{\prime}(i, i) \leftarrow w^{\prime}(i, i)+w^{\prime}(i, j)-w(i, j) \cdot \frac{w^{\prime}(j, i)}{w(j, i)}\)
    \(w^{\prime}(i, j) \leftarrow w(i, j) \cdot \frac{w^{\prime}(j, i)}{w(j, i)}\)
end if
```

After this modification we add again the random surfer which leads us to the new edge weight $f(i, j)$ :

$$
f(i, j) \leftarrow d \cdot w^{\prime}(i, j)+\frac{(1-d)}{n}
$$

We call our new graph $F$ with the edge weights $f$. The PageRank of Equality is then $\mathrm{pr}_{F}$ of the graph $F(M, f)$.

### 3.2.3 Model 3 - Knowing

The third variant is called Knowing. This variant gives edges a higher weight when the teams on the nodes have played more often against each other. We rate them higher because we say that those results are more confident.

We multiply the edges $w(i, j)$ with the number of matches between $i$ and $j$ and add the random surfer:

$$
w^{\prime \prime}(i, j) \leftarrow w(i, j) \cdot\left(\sum_{m(i, j) \in M} 1\right)
$$

We scale the edge weights $w^{\prime \prime}(i, j)$ and add a random surfer. The new edge weight is $g$ :

$$
g(i, j) \leftarrow d \cdot \frac{w^{\prime \prime}(i, j)}{\sum_{k \in T} w^{\prime \prime}(i, k)}+\frac{(1-d)}{n}
$$

The PageRank of Knowing is then $\operatorname{pr}_{G}$ of the graph $G(M, g)$.


Figure 3.1: Graph with the matches between Lausanne (L), Thun (T) and Young Boys (Y)

### 3.3 Example

We have in the previous Section introduced our three variants for the calculation of the PageRank. We make now an example with three teams: Lausanne, Thun and the Young Boys.

We use for a victory $v=2$, a tie $t=1$ and for the random surfer $d=0.9$. According to the matches in Table 3.1 we can create the simple graph $D$ which is in Figure 3.1.

We can write the matrix with the adjacency matrix $Q=\left(\begin{array}{lll}0 & 0 & 2 \\ 2 & 0 & 1 \\ 2 & 3 & 0\end{array}\right)$ which describes the graph $D$.

Table 3.1: Table with matches between Lausanne, Thun and the Young Boys

| Team 1 | Team 2 | Score Team 1 | Score Team 2 |
| :---: | :---: | :---: | :---: |
| Lausanne | Thun | 1 | 0 |
| Thun | Young Boys | 1 | 1 |
| Young Boys | Lausanne | 0 | 1 |
| Young Boys | Thun | 1 | 2 |
| Young Boys | Lausanne | 3 | 0 |



Figure 3.2: Scaled graph with the matches between Lausanne (L), Thun (T) and Young Boys (Y)

### 3.3.1 Variant 1 - Normal

The matrix $S=\left(\begin{array}{ccc}0 & 0 & 1 \\ 0.667 & 0 & 0.333 \\ 0.4 & 0.6 & 0\end{array}\right)$ can be directly written down from the graph $E$ in Figure 3.2.

In a next step we calculate $P=d \cdot S+\frac{(1-d)}{n} \cdot A$.
$P=d \cdot\left(\begin{array}{ccc}0 & 0 & 1 \\ 0.667 & 0 & 0.333 \\ 0.4 & 0.6 & 0\end{array}\right)+\frac{0.1}{3} \cdot\left(\begin{array}{lll}1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1\end{array}\right)$
We calculate the PageRank $\mathrm{pr}_{E}$. The Young Boys are first with 0.4107. Lausanne has a PR of 0.3342 and is on place 2. Thun is last with the smallest PR 0.2551.

### 3.3.2 Model 2 - Equality

Again, we start with the graph $D$ in Figure 3.1. We scale him and get $E$ in Figure 3.2.

Now we have to compare the weights of the edges in $E$ when there is one in direction $(i, j)$ and one in $(j, i)$. That is the case between Lausanne and Young Boys and between Thun and Young Boys.
First we compare the edges between Lausanne and Young Boys: $\frac{1}{2}>\frac{0.4}{2}$. Therefore $w^{\prime}(L, L)=0+1-2 \cdot \frac{0.4}{2}=0.6$ and $w^{\prime}(L, Y)=2 \cdot \frac{0.4}{2}=0.4$.
Now we adjust the weights between Thun and Young Boys: $\frac{0.333}{1}>\frac{0.6}{3}$. Thus, $w^{\prime}(T, T)=0+0.333-1 \cdot \frac{0.6}{3}=0.133$ and $w^{\prime}(T, Y)=1 \cdot \frac{0.6}{3}=0.2$.
We can create now the graph in Figure 3.3.
In a last step we add the random walker and have now the resulting graph $F$.


Figure 3.3: Modified scaled graph with the matches between Lausanne (L), Thun (T) and the Young Boys (Y)

We calculate with graph $F$ the PageRank pr ${ }_{F}$. Compared to Normal, Lausanne and the Young Boys have changed the places. Lausanne is first with a PR of 0.5428 , second are the Young Boys with 0.2417 and Thun is still last with 0.2156 .

### 3.3.3 Model 3 - Knowing

At the beginning we have again graph $D$. We multiply the weight of the edge $i, j$ with the number of matches between $i$ and $j$. The already new scaled graph is in Figure 3.4. In a further step we add a random surfer. Now we have the graph $G$ and calculate $\mathrm{pr}_{G}$. Again, Thun has the smallest PageRank with 0.2467 . Lausanne is on place 2 like in the Normal model with 0.3067 . The highest PageRank have the Young Boys with 0.4285 .


Figure 3.4: Scaled graph modified with Knowing with the matches between Lausanne (L), Thun (T) and the Young Boys (Y)

### 3.4 Prediction Models

To test our rankings, we predict matches. To make a prediction we use four different models which are explained in this section.

On day $c_{1}$ plays team $i$ against $j$. One day before the match between team $i$ and $j$, on day $c_{0}$, we have a graph $Z_{0}\left(M_{0}, z\right)$. The graphs from our three variants $E(M, e), F(M, f)$ or $G(M, g)$ can be the graph $Z_{0}\left(M_{0}, z\right)$. The PageRank of team $i$ is $\operatorname{pr}_{Z_{0}}(i)$ and the PageRank of team $j$ is $\operatorname{pr}_{Z_{0}}(j)$ with respect to the graph $Z_{0}$.

Let $M_{1}=M_{0} \oplus m\left(\mathrm{sc}_{i}>\mathrm{sc}_{j}, c_{1}\right)$ be all matches before day $c_{1}$ plus a victory of team $i$ on day $c$, and let be $M_{2}=M_{0} \oplus m\left(\mathrm{sc}_{i}<\mathrm{sc}_{j}, c_{1}\right)$. The graph $Z_{1}\left(M_{1}, z\right)$ represents the graph by a victory of team $i$. The new PageRank of team $i$ is given by $\operatorname{pr}_{Z_{1}}(i)$ with respect to the graph $Z_{1}\left(M_{1}, z\right)$. The graph $Z_{2}\left(M_{2}, z\right)$ shows the new graph by a victory of team $j$ on day $c_{1}$ and the new PageRank of team $j$ is in this case $\operatorname{pr}_{Z_{2}}(j)$.

Let $\Delta \operatorname{pr}(i)$ be the PageRank raise for team $i$ in case of a victory of $i$.

$$
\Delta \operatorname{pr}(i) \leftarrow \operatorname{pr}_{Z_{1}}(i)-\operatorname{pr}_{Z_{0}}(i)
$$

The PageRank raise for team $j$ with a victory is $\Delta \operatorname{pr}(j)$ :

$$
\Delta \operatorname{pr}(j) \leftarrow \operatorname{pr}_{Z_{2}}(j)-\operatorname{pr}_{Z_{0}}(j)
$$

### 3.4.1 PageRank Difference (PRD)

The first model PRD considers the PageRanks of team $i$ and $j$ on the day $c_{0}$ before the match.

$$
\begin{gathered}
\operatorname{pr}_{Z_{0}}(i) \geq \operatorname{pr}_{Z_{0}}(j) \Rightarrow \text { The ranking predicts a victory of team } i . \\
\operatorname{pr}_{Z_{0}}(i)<\operatorname{pr}_{Z_{0}}(j) \Rightarrow \text { The ranking predicts team } j \text { as winner. }
\end{gathered}
$$

Hence we bet on the team with the higher PageRank before the match.

We will later use the definition PageRank difference again: The PageRank difference is generally $\left|\operatorname{pr}_{Z_{0}}(i)-\operatorname{pr}_{Z_{0}}(j)\right|$.

### 3.4.2 Winning Probability Difference (WPD)

A second model, the WPD, compares the growth of the PageRanks of both teams in case of a victory. It calculates the winning probabilities of teams $i$ and $j$. They
are based on how much the PageRank of both teams raises if team $i$ respectively team $j$ would win.

$$
\begin{aligned}
& \frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)} \geq 0.5 \Rightarrow \text { The ranking predicts a victory of team } i . \\
& \frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}<0.5 \Rightarrow \text { The ranking predicts team } j \text { as winner. }
\end{aligned}
$$

The term winning probability difference is defined with $\left|\frac{\Delta \operatorname{pr}(i)-\Delta \operatorname{pr}(j)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}\right|$.
Model WPD and PRD do not make the same predictions. It is possible that the PageRank of the higher ranked team can grow more than the PR of a lower ranked team. It depends against which teams they have played before and therefore are connected with which teams in the graph.

### 3.4.3 Randomly Winning Probability (RWP)

This model uses again the winning probabilities from Section 3.4.2.
The difference to the WPD model is that we bet with RWP with a probability $\frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}$ on team $i$. In the other case we bet on a win of team 2. Consequently we do not bet with this model in all matches on the team with the higher winning probability.

### 3.4.4 Tie Prediction

## Tie Prediction with PRD

This prediction model tries to predict ties as well. We set at the beginning a specified PageRank difference $\varepsilon$.

$$
\left|\operatorname{pr}_{Z_{0}}(i)-\operatorname{pr}_{Z_{0}}(j)\right| \leq \varepsilon \Rightarrow \text { The ranking predicts a tie. }
$$

$$
\begin{aligned}
& \mathrm{pr}_{Z_{0}}(i)-\mathrm{pr}_{Z_{0}}(j)>\varepsilon \Rightarrow \text { The ranking predicts team } i \text { as winner. } \\
& \operatorname{pr}_{Z_{0}}(i)-\mathrm{pr}_{Z_{0}}(j)<-\varepsilon \Rightarrow \text { The ranking predicts a win of team } j .
\end{aligned}
$$

## Tie Prediction with WPD

We can do the same with the WPD model. Again, we set first a certain winning probability difference $\zeta$ between team $i$ and $j$.

$$
\left|\frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}-0.5\right| \leq \zeta \Rightarrow \text { The ranking predicts a tie. }
$$

$$
\begin{aligned}
& \frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}+\zeta>0.5 \Rightarrow \text { The ranking predicts team } i \text { as winner. } \\
& \frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}+\zeta<0.5 \Rightarrow \text { The ranking predicts a win of team } j .
\end{aligned}
$$

### 3.4.5 Parameters for Predictions

We have two additional parameters when we make the predictions. We do not want to consider teams which have not played a lot matches inside the time span $\lambda$.

## Minimum Matches

The ranking gets falsified with teams which have played only a few matches. Edges are created with matches. If a team has not played often, it will not have a lot of edges and is not well connected to the other nodes. Thus, we define a minimum number of matches $N$ in a time span $\lambda$ which a team has to have completed before its results will be included in the PageRank calculation.

## Tolerance

The tolerance $\alpha$ is a second previously set factor to filter matches where the new PageRank of a team raises disproportionally high in case of a victory.
Let $Z_{0}=G\left(M_{0}, z\right)$ be the graph before the match between team $i$ and $j$. The PageRank of team in the graph $Z_{0}$ is defined with $\mathrm{pr}_{Z_{0}}(i)$ for team $i$. When team $i$ wins, the new graph is defined with $Z_{1}=G\left(M_{0} \oplus m\left(\mathrm{sc}_{i}>\mathrm{sc}_{j}\right), z\right)$. The PageRank of team $i$ after a victory is $\mathrm{pr}_{Z_{1}}(i)$.

$$
\left|\frac{\mathrm{pr}_{Z_{1}}(i)-\mathrm{pr}_{Z_{0}}(i)}{\operatorname{pr}_{Z_{0}}(i)}\right|>\alpha \rightarrow \text { The ranking does not predict this match. }
$$

We only use $\alpha$ for the league seasons. We do not use it for the FIFA matches. It was difficult to find a reasonable $\alpha$ for those matches because it often filtered the World Cup matches with high weights. For us it is important to predict those matches.

### 3.5 Respecting Home/Away

The home team has won in $49.30 \%$ of the matches in the Bundesliga between 1975 and 2014 . Only in $24.40 \%$ the away team could win. This high home win rate is impressing and we have to use this fact. We made four different models which respect the home advantage. We have two main ideas. In the first we give
different points for home and away results. The second says that the home team wins even if it has a lower PageRank or lower winning probability.

### 3.5.1 Homebonus

This model rates away ties or away wins with the same amount of points as for the home team. The PageRank calculation is the same as without respecting the home advantage. But during making the predictions the model adds a bonus to the team. The method works similar as the tie prediction in Section 3.4.4. This bonus is defined with $\rho$ for the prediction models with PRD and $\sigma$ for the models with WPD. The Homebonus changes the PRD model to:

$$
\begin{aligned}
\tau & =i \text { and } \operatorname{pr}_{Z_{0}}(i)+\rho \geq \operatorname{pr}_{Z_{0}}(j) \Rightarrow \text { The ranking predicts a victory of team } i . \\
\tau & =i \text { and } \operatorname{pr}_{Z_{0}}(i)+\rho<\operatorname{pr}_{Z_{0}}(j) \Rightarrow \text { The ranking predicts team } j \text { as winner. }
\end{aligned}
$$

The prediction model for WPD Homebonus is defined as:

$$
\begin{aligned}
\tau & =i \text { and } \frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}+\sigma \geq 0.5 \Rightarrow \text { The ranking predicts a victory of team } i . \\
\tau & =i \text { and } \frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}+\sigma<0.5 \Rightarrow \text { The ranking predicts team } j \text { as winner. }
\end{aligned}
$$

If there is no identified home team $(\tau=0)$, we use the usual PRD and WPD model in Section 3.4.1 and Section 3.4.2.

### 3.5.2 Pluspoint

As already mentioned before, we expect in four matches almost two home wins, one tie and one away win. With this fact we made another distribution of the points. But we do not want to add more points into the system with four matches with a home team compared to four matches in a neutral place. If the match is on a neutral place, victory and tie are still rated with $v$ for a win and $t$ for a tie for both teams. But if there exists a home team we use the two rules from before. A home win is rated with $0.75 \cdot v$, an away victory with $1.5 \cdot v$. A home tie with $\frac{2}{3} \cdot t$ and an away tie with $\frac{4}{3} \cdot t$. This model is called Pluspoint:

$$
s_{i} \leftarrow \begin{cases}v & \text { if } \tau=0 \text { and } m\left(\mathrm{sc}_{i}>\mathrm{sc}_{j}\right) \\ \frac{3}{4} \cdot v & \text { if } \tau=i \text { and } m\left(\mathrm{sc}_{i}>\mathrm{sc}_{j}\right) \\ \frac{3}{2} \cdot v & \text { if } \tau=j \text { and } m\left(\mathrm{sc}_{i}>\mathrm{sc}_{j}\right) \\ t & \text { if } \tau=0 \text { and } m\left(\mathrm{sc}_{i}=\mathrm{sc}_{j}\right) \\ \frac{2}{3} \cdot t & \text { if } \tau=i \text { and } m\left(\mathrm{sc}_{i}=\mathrm{sc}_{j}\right) \\ \frac{4}{3} \cdot t & \text { if } \tau=j \text { and } m\left(\mathrm{sc}_{i}=\mathrm{sc}_{j}\right) \\ 0 & \text { else }\end{cases}
$$

### 3.5.3 Combo

This model combines the two models from Section 3.5.1 and Section 3.5.2 and is called Combo. We use again the new point distribution for matches with a home team. The graph for this model is built with this point distribution.

$$
s_{i} \leftarrow \begin{cases}v & \text { if } \tau=0 \text { and } m\left(\mathrm{sc}_{i}>\mathrm{sc}_{j}\right) \\ \frac{3}{4} \cdot v & \text { if } \tau=i \text { and } m\left(\mathrm{sc}_{i}>\mathrm{sc}_{j}\right) \\ \frac{3}{2} \cdot v & \text { if } \tau=j \text { and } m\left(\mathrm{sc}_{i}>\mathrm{sc}_{j}\right) \\ t & \text { if } \tau=0 \text { and } m\left(\mathrm{sc}_{i}=\mathrm{sc}_{j}\right) \\ \frac{2}{3} \cdot t & \text { if } \tau=i \text { and } m\left(\mathrm{sc}_{i}=\mathrm{sc}_{j}\right) \\ \frac{4}{3} \cdot t & \text { if } \tau=j \text { and } m\left(\mathrm{sc}_{i}=\mathrm{sc}_{j}\right) \\ 0 & \text { else }\end{cases}
$$

When we make the predictions, we add a little bonus $\rho$ for the home team in the PRD model and $\sigma$ for the home team in the WPD model:

$$
\begin{aligned}
& \operatorname{pr}_{Z_{0}}(i)+\rho \geq \operatorname{pr}_{Z_{0}}(j) \Rightarrow \text { The ranking predicts a victory of team } i . \\
& \operatorname{pr}_{Z_{0}}(i)+\rho<\operatorname{pr}_{Z_{0}}(j) \Rightarrow \text { The ranking predicts team } j \text { as winner. } \\
& \frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}+\sigma \geq 0.5 \Rightarrow \text { The ranking predicts a victory of team } i . \\
& \frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}+\sigma<0.5 \Rightarrow \text { The ranking predicts team } j \text { as winner. }
\end{aligned}
$$

### 3.5.4 Model with home and away Team for each Team (TwoTeam)

The difference to the models before is that each team is modeled with two teams, a home team and an away team. We use this model only for league matches because there is in each match a home team. A second fact of the league matches is that the teams have a lot more matches which leads to more edges between the nodes in the graph.

Let $M=\left(\left(i, j, \mathrm{sc}_{i}, \mathrm{sc}_{j}, c, \Gamma, \tau\right), \ldots\right)$ be the sequence of matches in a league. In the set $T$ are all teams. The size of $T$ is $n$. Again, $i$ and $j$ are the teams of a match $m$. The graph that represents $M$ has $2 \cdot n$ nodes. The node $i$ in the graph represents the home team and node $2 i$ represents the away team of a team.

We use $\tau$ in $m=\left(i, j, \mathrm{sc}_{i}, \mathrm{sc}_{j}, c, \Gamma, \tau\right)$ with

$$
\tau \leftarrow \begin{cases}i & \text { if } i \text { is hometeam } \\ j & \text { if } j \text { is hometeam }\end{cases}
$$

The points $h$ are calculated with:

```
if \(\tau \neq i\) then
    \(h_{2 i}(m) \leftarrow s_{i}\)
else
    \(h_{i}(m) \leftarrow s_{i}\)
end if
```

The weight of an edge $(i, 2 j)$ is the sum of all points the team $2 j$ has earned in matches against $i$ when $i$ was the home team:

$$
w(i, 2 j) \leftarrow \sum_{m \in M(c \in \lambda)} h_{2 j}
$$

We can again create the resulting graph with the example from Section 3.3 in Figure 3.5. We assume that the first called team has always home advantage.

With this model each team has two PageRanks. It is difficult to say, what the overall PageRank of a team is (home and away team). A better statement is to compare only the home and the away strengths and to find on this way the best home team respectively away team.


Figure 3.5: Graph with the matches between Lausanne (home team=LH, away team=LA), Thun (home team $=\mathrm{TH}$, away team=TA) and the Young Boys (home team $=\mathrm{YH}$, away team $=\mathrm{YA}$ ) with the TwoTeam model

## Fetching Matches

We need results of football matches to make our predictions. To get and store those informations, we use scrapy and mysql.

### 4.1 League

The source of the League matches is http://www.fussballdaten.de/. We have chosen Bundesliga (1975-2014), Ligue 1 (2002-2014) Premier League (2002-2014), Primera Division (2002-2014) and Serie A (2002-2914) for testing our algorithm. We stored the date, the name of both teams, the final result and the match day of each match.

### 4.2 FIFA Matches

The second part of matches we wanted are the international matches since 1872. The source to fetch them was the official website of the FIFA. We only fetched matches which are played between official members of FIFA. We stored for each match the date, both team names, the competition of the match, the place (city) and the result including information about half-time score and extra-time and penalty shoot-out (if it happened).
The competition declares the game type. There are six different cases in all FIFA matches: friendly, continental qualifier, World Cup qualifier, continental final, Confederations Cup, World Cup final.

The match places are located in the correspondent country with the help of www.maxmind.com. We needed to do this to detect the home team in a match. This association is based on the belonging of cities to countries in May 2014. A problem is that country borders have changed during the years since 1872. E.g. a match of the Soviet Union in Moscow in 1970 is rated as match in a neutral place because Moscow is associated to Russia.

Another problem is that some city names exist in more than one country. To associate the correct country to the place we used the informations in our database. The number of matches of a team in those places helped to locate the city in the correct country.

### 4.3 FIFA/Coca-Cola Ranking

We want to compare our ranking before and after a World Cup with the official FIFA/Coca-Cola Ranking. Those rankings are crawled too. We grabbed the informations from http://www.fifa.com/worldranking/. The different data elements are the date of the ranking, team names, positions, points and changes of position and changes of points since the last published ranking.

## Testing

The best way to test if a ranking is good is to predict matches[8]. We therefore predict with our different models matches and look how many correct predictions we make with them.

In this chapter we set our different parameters. We adjust them with the information of correct predictions of the Bundesliga seasons 1975-2014 and FIFA matches between 1900 and 1998. The best parameters are where we predict most matches correct. Therefore, we always vary one parameter and let the others fix and plot the correct prediction rate.

A general thing concerns the random surfer parameter $d$. The results showed that $d=0.9$ was in most cases the best choice and the prediction rates did not changed much by varying $d$ between 0.8 and 1.0. Thus, we set for the following testings always $d=0.9$. Google uses for the search engines a random surfer parameter equals 0.85 which showed in their testings the best results[9].

### 5.1 League matches

### 5.1.1 Homebonus

We first vary the time span $\lambda$. It describes the time in which matches are considered and included in the ranking. We chose half year steps as minimum step size. We can see in Figure 5.1 that a time span of 2.5 years makes the best predictions with a rate of $51.53 \%$ with the PRD Normal model.
Already in the first tests and until the end it has shown that PRD Equality and WPD Normal work the best for Homebonus. Thus, we will focus on these two models. But we will always make predictions with the other four models too. Perhaps they can significantly improve their predictions by varying the parameters.

First we look at PRD Equality. We made predictions for various $\rho$ and various


Figure 5.1: Correct predictions for Bundesliga seasons 1975-2014 with Homebonus and different $\lambda$
relations of $v / t$. The bonus $\rho$ is added to the PageRank of the home team when making the predictions with PRD. The results are in Figure 5.2. The best predictions are made with $v / t$ around 1.8. In a further step we made smoother steps

Bundesliga 1975-2014 - PRD Equality ( $\alpha=0.18, N=10, \lambda=2.5$ )


Figure 5.2: Correct Predictions for Bundesliga seasons 1975-2014 with Homebonus PRD Equality and different $\rho$
for $\rho$ and the relation $v / t$. The highest prediction rate is found with $\rho=0.030$ and $v / t=1.8$ in Figure 5.3.
After adjusting $\rho$ we tried to find the best $v / t$ in Figure 5.4. We made the best predictions with $\rho=0.030, v / t=1.82$, and $\lambda=2.5$ with PRD Equality.


Figure 5.3: Correct predictions for Bundesliga seasons 1975-2014 with Homebonus PRD Equality and different $\rho$

Those parameters lead to $51.83 \%$ correct predictions for the Bundesliga seasons between 1975 and 2014.


Figure 5.4: Correct predictions for Bundesliga seasons 1975-2014 with Homebonus PRD Equality and different $v / t$

Now we set the parameters for WPD Normal. In a first step we try different $v / t$ and $\sigma$. When making predictions with WPD, $\sigma$ defines the bonus which is added
to the winning probability of the home team. It arises in Figure 5.5 and Figure 5.6 that $v / t$ higher than 2.4 is a good choice with $\sigma$ around 0.30 .


Figure 5.5: Correct predictions for Bundesliga seasons 1975-2014 with Homebonus WPD Normal and different $v / t$ and $\sigma$


Figure 5.6: Correct predictions for Bundesliga seasons 1975-2014 with Homebonus WPD Normal and differrnt $v / t$

We make again predictions with $v / t$ around 2.5 and a $\sigma=0.30$. The outcomes are in Figure 5.6. We choose $v / t=2.51$ for further calculations because the
prediction rate with $v / t=2.51$ has the smallest standard deviation comparing with the rates of $v / t=2.50$ and $v / t=2.52$.

We try to get better predictions with $v / t=2.51$ and $\sigma$ around 0.30 . Figure 5.7 shows that we make the best predictions with $\sigma=0.30$. Finally we predict with WPD Normal Homebonus $\sigma=0.3, v / t=2.51$ and $\lambda=2.551 .66 \%$ of the matches correct.

Bundesliga 1975-2014 - WPD Normal Homebonus ( $\alpha=0.18, \mathrm{~N}=10, \mathrm{v} / \mathrm{t}=2.51, \lambda=2.5$ )


Figure 5.7: Correct predictions for Bundesliga seasons 1975-2014 with Homebonus WPD Normal and different $\sigma$

### 5.1.2 Pluspoint

The first parameter setting concerns the time span $\lambda$. We predict again with an initial $v / t=3.0$ and the random surfer parameter $d=0.9$ matches of the Bundesliga between 1975 and 2014. The results are shown in Figure 5.8.

Again, we choose a time span $\lambda=2.5$ which seems to be the best choice because we produce the highest prediction rates with it. In a next step we wanted to optimize $v / t$. It was quite difficult because the prediction rates were almost constant for a $v / t$ between 1.8 and 3.4. That is a big difference compared with the Homebonus results where the predictions for different $v / t$ had a real peak somewhere. The highest prediction rates are made with $v / t$ around 3 . In further testings we make the best predictions with $v / t=3.06$ and WPD Normal.

Figure 5.8 and 5.9 show that the WPD models make correct predictions around


Figure 5.8: Correct predictions for Bundesliga seasons 1975-2014 with Pluspoint for $v / t=3.0$ and different $\lambda$


Figure 5.9: Correct predictions for Bundesliga seasons 1975-2014 with Pluspoint for $\lambda=2.5$ and different $v / t$
$51 \%$. But the PRD models make correct predictions in only $46.5 \%$ of the matches. It seems that the different points for home/away matches result in a similar PageRank ranking table. But the winning probabilities are really different to models without respecting the home advantage. The WPD models are calculated with the winning probabilities. Usually a team can raise the PageRank with an
away win more because an away win gives a higher edge weight. Therefore, the WPD for home teams is higher as for away teams. This helps to predict more home wins.

### 5.1.3 Neutral

We try to set the time span $\lambda$ first. But in this case it is harder to find the best $\lambda$ as you can see in Figure 5.10. We decided to choose $\lambda=2.5$ and $\lambda=3$.


Figure 5.10: Correct predictions for Bundesliga seasons 1975-2014 with Neutral and different $\lambda$ with $v / t=2.0$

We tune the parameters with $\lambda=3$. In Figure 5.11 we see that PRD Equality is clearly better than all the other models. The best predictions are made with a small difference between $v$ and $t$. The best $v / t$ for PRD Equality is 1.20 with a correct prediction rate of $46.54 \%$.

We know from the previous sections that $\lambda=2.5$ years is a good choice for the time span. We tune $v / t$ again. We see in Figure 5.12 that PRD Equality works fine and PRD Normal too. Because we have already optimized PRD Equality in Figure 5.11 and it does not make better predictions than $46.54 \%$, we focus on PRD Normal.

In Figure 5.12 we see the peak of correct predictions are done with $v / t=2.35$. The correct prediction rate is there $46.24 \%$.


Figure 5.11: Correct predictions for Bundesliga seasons 1975-2014 with Neutral and different $v / t$ with $\lambda=3$


Figure 5.12: Correct predictions for Bundesliga seasons 1975-2014 with Neutral and different $v / t$ with $\lambda=2.5$

### 5.1.4 Random bet on Winning Probability - RWP

To test the RWP prediction model we take the parameters from WPD Normal Homebonus and compare them with the results from Section 5.1.1. We choose this model because there resulted $51.66 \%$ WPD. The time span $\lambda=2.5, v / t=2.51$
and $\sigma=0.3$. When we test RWP on the Bundesliga seasons 1975-2014, we get a correct prediction rate of $47.33 \%$ for WPD Normal. This rate is around $5 \%$-points lower than the rate with the WPD Normal Homebonus.


Figure 5.13: Plot of winning probability differences versus the number of matches in percentage for WPD Normal with RWP Homebonus

Why is the prediction rate so much lower? In Figure 5.13 on the x -axis are the winning probability differences. They are separated in sectors. Each sector is 0.025 long. Remember, we defined the winning probability difference with $\left|\frac{\Delta \operatorname{pr}(i)-\Delta \operatorname{pr}(j)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}\right|$. On the y-axis are the number of matches in percentage which have the corresponding winning probability difference. We have for each sector three bars. The first bar describes the correct predictions we made with WPD Normal Homebonus in combination with RWP. The second bar describes the matches where we predicted a winner but the match ended with a tie. The third bar shows the number of matches where we predicted the wrong winner. The second and third bar are wrong predictions of our model.
We compare it with the same sort of plotting but with the results of WPD Normal Homebonus in Figure 5.14. With this model we always bet on the team with the higher winning probability. We see that WPD Normal Homebonus makes clearly better predictions if we always bet on the team with the higher winning probability. A second thing we can see in both figures are that the majority of the matches have small winning probability differences. The RWP model decides with the winning probability of team $i$ to predict a win of team $i$. Therefore RWP changes often the team if the winning probability is only less higher than $50 \%$. If the majority of the matches have a winning probability difference between 0 and 0.2 , RWP changes too often the team. Therefore is the WPD Normal Homebonus RWP model so worse compared with the simple WPD Normal Homebonus model.


Figure 5.14: Plot of winning probability differences versus the number of matches in percentage for WPD Normal Homebonus

Thus, we do not pursue to make predictions with the RWP model.

### 5.1.5 Tie Prediction

This model predicts wins and ties. The idea is that it predicts a tie if the PageRank difference is in a certain range. This range is called $\varepsilon$ when we use PRD.

$$
\left|\operatorname{pr}_{Z_{0}}(i)-\operatorname{pr}_{Z_{0}}(j)\right| \leq \varepsilon \Rightarrow \text { The ranking predicts a tie. }
$$

$$
\operatorname{pr}_{Z_{0}}(i)-\operatorname{pr}_{Z_{0}}(j)>\varepsilon \Rightarrow \text { The ranking predicts team } i \text { as winner. }
$$

$$
\operatorname{pr}_{Z_{0}}(i)-\operatorname{pr}_{Z_{0}}(j)<-\varepsilon \Rightarrow \text { The ranking predicts a win of team } j \text {. }
$$

For the prediction using the winning probabilities (WPD) we bet on a tie if the winning probability difference is in $\zeta$.

$$
\begin{gathered}
\left|\frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}-0.5\right| \leq \zeta \Rightarrow \text { The ranking predicts a tie. } \\
\frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}+\zeta>0.5 \Rightarrow \text { The ranking predicts team } i \text { as winner. } \\
\frac{\Delta \operatorname{pr}(i)}{\Delta \operatorname{pr}(i)+\Delta \operatorname{pr}(j)}+\zeta<0.5 \Rightarrow \text { The ranking predicts a win of team } j .
\end{gathered}
$$



Figure 5.15: Plot of PageRank differences versus the number of matches in percentage for PRD Normal Neutral

Tests have shown that we make worse predictions when using this Tie Prediction model. We can see the reason in Figure 5.15. On the x -axis in this figure are the PageRank difference of the two teams. The $y$-axis shows how many matches had the corresponding PageRank difference on the x -axis. There are again three bars for each PageRank difference region. The first indicates the correct predictions. The second shows the wrong predictions when the true result was a tie. And the third is for the matches where our model predicted the wrong winner.

We see in Figure 5.15 that there is no PageRank difference region where more matches end with a tie than with a win. So, we can not improve our predictions when we would choose an $\varepsilon>0$. We will make for all $\varepsilon>0$ worse predictions than without using this $\varepsilon$.
We have the same problem for $\zeta$. Figure 5.14 shows that there does not exist a winning probability difference where it would be better to bet on a tie. Therefore the best predictions are done if we do not try to predict ties. We only bet on a winner.

### 5.1.6 Combo

The Combo Model is a combination of Homebonus and Pluspoint. We test it on the time span $\lambda=2.5$ which led us to good results for Homebonus and Pluspoint. We choose for the first tests $v / t=2.0$.

The results are in Figure 5.16 for PRD and in Figure 5.17 for WPD. We see that


Figure 5.16: Correct predictions for Bundesliga seasons 1975-2014 with Combo PRD and different $\rho$
we get good results for the WPD when we put $\sigma$ near to zero. But Combo is in this case the simple Pluspoint model. Therefore we concentrate on the PRD models. We try different $\rho$ with $v / t=1.5$.


Figure 5.17: Correct predictions for Bundesliga seasons 1975-2014 with Combo WPD and different $\sigma$

The highest correct prediction rate is done with $v / t=1.5$ and $\rho=0.035$. Thus,


Figure 5.18: Correct predictions for Bundesliga seasons 1975-2014 with Combo PRD and different $\rho$
we take this $\rho$ and vary $v / t$ in Figure 5.19. Combo PRD Equality makes the best


Figure 5.19: Correct predictions for Bundesliga seasons 1975-2014 with Combo PRD and different $v / t$
predictions with $\rho=0.035$ and $v / t=1.56$. In $52.08 \%$ of the matches we predict the correct result.

### 5.1.7 TwoTeams Neutral

In this model each team is represented by two nodes. We test it first in combination with the Neutral model. We do not respect the home/away team.

We assume from the previous results that a time span of $\lambda=2.5$ is the best choice and predict matches for different $v / t$. The best predictions are done with


Figure 5.20: Correct predictions for Bundesliga seasons 1975-2014 with Two Teams Neutral and different $v / t$

WPD Normal with $v / t=2.66$ and PRD Equality with $v / t=2.54$. The rates are $50.32 \%$ and $50.31 \%$. When we compare those rates with the results from Section 5.1.3 we see that we make better predictions with the TwoTeams model than with the Neutral single-team model.

### 5.1.8 TwoTeams Pluspoint

We predict this time matches with the combination of the Two Teams and Pluspoint. The Pluspoint gives different number of points for home and away results. We use again the time span $\lambda=2.5$ and the random surfer parameter $d=0.9$. The best model is WPD Normal with $v / t=1.05$. This model predicts $50.54 \%$ of the matches correct. We see again that the PRD and Homebonus models do not work well together. The prediction rates of the PRD models are worse than of the WPD models.


Figure 5.21: Correct predictions for Bundesliga seasons 1975-2014 with TwoTeams Pluspoint model and different $v / t$

### 5.2 FIFA matches

To set the parameters for the international matches, we use the matches between 1900 and 1998. We do not predict friendlies. We only predict matches of a tournament (qualifier and final matches). Friendly matches are often used by teams to test new players or systems and do mostly not reflect the real strength of a team. But we use them to create the graph because we need as much matches as possible. We hope that those matches give us anyway important information about the strengths of the teams. The majority of the matches in our database are friendlies therefore we have use them.
The FIFA matches after 1998 are used to see how good our ranking is. (Again, we use all matches to create the graph but we do not predict friendlies.)

### 5.2.1 Neutral

The big difference of the Neutral model is that it does not consider the home advantage. We start with the time span $\lambda$ of four years, the random surfer parameter $d=0.9$ and the relation $v / t=3.0$. The match status factor is always the classic version $\gamma=[4,3,2.5,1]$.

Figure 5.22 shows the results. A good choice are small values for $\beta$ which is used to calculate the time degradation.

A second plot shows how the prediction rates look like when we use a fix $\beta=1.4$


Figure 5.22: Correct predictions for FIFA matches 1900-1998 with Neutral and $\lambda=4$ for different $\beta$
and vary $v / t$. The plot is in Figure 5.23. The best prediction rates for FIFA


Figure 5.23: Predictions for FIFA matches (1900-1998) with Neutral and different $v / t$
matches are around $55.6 \%$ and are done with PRD Normal.
The next change of parameter is the time span. We set now $\lambda$ to 8 . We predict again all FIFA matches and have a special look on the World Cup matches with
$v / t=2.0$ and different $\beta$.


WC 1934-1998 ( $\mathrm{N}=10, \mathrm{~d}=0.9, \lambda=8, v / t=2.0, \gamma=[4,3,2.5,1]$ )


Figure 5.24: Correct predictions for FIFA matches 1900-1998 with Neutral and $\lambda=8$ for different $\beta$

The results are in Figure 5.24. We can see that we make better predictions with a higher $\lambda$. We predict around $57.5 \%$ of the matches correct with PRD Equality when we use the FIFA matches. With WPD Equality we reach $56.80 \%$ correct predictions during the World Cups.

In Figure 5.25 are the results when we vary $v / t$ for $\beta=1.6$.
Before we go into further details we try first the models which respect the home advantage. Section 5.1 has shown that we can improve our results around $5 \%$ points when we use the Homebonus or the Pluspoint model.

### 5.2.2 Pluspoint

One big decision is the the time. How long should matches be considered? We start with four years like FIFA does. The other parameters for the first testing are time degradation $\beta=2$ and match status factors $\gamma=[4,3,2.5,1]$.

We vary only the relation $v / t$. The prediction rates are shown in Figure 5.26. The highest prediction rate for correct FIFA matches makes WPD Normal with $58.56 \%$ and a $v / t=1.2$. WC matches are predicted in $52.28 \%$ correct with WPD Normal again and a relation $v / t=1.4$.
In the next Figure 5.27 are matches predicted with $v / t=1.2$ and different $\beta$.


Figure 5.25: Correct predictions for FIFA matches 1900-1998 with Neutral and $\lambda=8$ and $\beta=1.6$ for different $v / t$


Figure 5.26: Correct predictions for FIFA matches 1900-1998 with Pluspoint and $d=0.9, \lambda=4, \beta=2$ and $\gamma=[4,3,2.5,1]$

The best predictions are done with WPD Normal and $\beta=1.1$. We get $59.23 \%$ correct results for FIFA matches.

We will not go into detail for those parameters. We will change the time span first up to eight years. Again, we split the time span into four parts. Each part


Figure 5.27: Correct predictions for FIFA matches 1900-1998 with Pluspoint and different $\beta$
has a another time degradation factor calculated with $\beta$. First we vary $v / t$ and set $\beta=1.5$ and let the other parameters as before: $d=0.9$ and match status factors $\gamma=[4,3,2.5,1]$. We see in Figure 5.28 that WPD Normal and WPD Equality make around $60 \%$ correct predictions. The second thing is that we make now


Figure 5.28: Correct predictions for FIFA matches 1900-1998 with Pluspoint and different $v / t$
correct predictions in over $60 \%$ of the matches with $\lambda=8$. The predictions with
$\lambda=4$ for FIFA matches were $1 \%$-points lower than with $\lambda=8$. In World Cup matches the predictions with $\lambda=4$ were around $52 \%$. With $\lambda=8$ we predicted around $55 \%$ of the matches correct. Therefore we decide to predict matches with $\lambda=8$.

We focus on WPD Normal which makes the best predictions for all FIFA matches. But we will have an eye open during testing if WPD Equality can make the same or even higher correct prediction rates. We try now different $\beta$ in combination with different $v / t$ for WPD Normal. The results are in Figure 5.29.


Figure 5.29: Correct predictions for FIFA matches 1900-1998 with Pluspoint WPD Normal and different $\beta$ and $v / t$

It is strange but we predict more matches correct with a relation $v / t<1.0$. That would say that a victory gives less points than a tie. We do not want that. We will choose the best solution of $v / t \geq 1.0$.

We try now first different $\beta$ and $v / t$ in Figure 5.30. We make the best predictions with $\beta=1.5$ and $v=1.0$. when we look on the prediction rates for all FIFA matches and the World Cup matches. The predictions for the European Championship behave completely different than for the WC and FIFA matches.

Next, we adjust $v / t$ in smaller steps with $\beta=1.5$. The difference of the correct predictions with different $v / t$ around 1.0 is only a few matches. But we find in Figure 5.31 that the best predictions are done with $v / t=1.04$.

Until now we have not changed $\gamma$. We let $\gamma$ since the beginning on the classic version $[4,3,2.5,1]$. We have tried different combinations for $\gamma$ too. With


Figure 5.30: Correct predictions for FIFA matches 1900-1998 with Pluspoint WPD Normal and different $\beta$


Figure 5.31: Correct predictions for FIFA matches 1900-1998 with Pluspoint WPD Normal and different $v / t$
$\gamma=[4,3,2.5,2.5]$ we predict $60.53 \%$ FIFA matches correct (best rate is $60.63 \%$ with $\gamma=[4,3,3,2]$ ) and $56.08 \%$ of the WC matches correct (highest rate).

### 5.2.3 Homebonus

This model takes the home advantage into consideration. The difference to the Pluspoint is that it makes the same graph as the Neutral model. But it adds a little bonus to the home team when doing the predictions. The bonus for the home team with PRD prediction is $\rho$. When we predict matches with the help of winning probabilities, the bonus for the home team is there $\sigma$.

We did not really expect that this model works good for FIFA matches because in many matches there is no home team and especially during the finals of tournaments there is mostly no home team. The consequence is that we predict many matches, especially during the finals, like the Neutral model.

We set the time span from beginning to $\lambda=8$ because we have seen with the Neutral and the Pluspoint model that a higher time span causes better predictions. We first vary different $\rho$ and $\sigma$. The other parameters we use here are $v / t=2.0$, time degradation $\beta=1.5$, match status factor $\gamma=[4,3,3,2])$ and the random surfer parameter $d=0.9$.



Figure 5.32: Correct predictions for FIFA matches 1900-1998 with Homebonus for different $\sigma$ and $\rho$

We can see that the WPD models work really well. WPD Normal makes with $\rho=0.40$ in $61.46 \%$ of the FIFA matches correct prediction. If we only consider the World Cup matches, WPD Equality works the best. The correct predictions of WPD Equality are $57.89 \%$ for $\rho$ around 0.40 . Even a PRD model, PRD Equality, predicts almost $60 \%$ of the FIFA matches correct.

In a next step we will vary $\beta$ and $v / t$ with $\rho=0.42$ and $\sigma=0.0045$. First we
let $\beta=1.5$ and predict matches for different $v / t$. The results are in Figure 5.33


Figure 5.33: Correct predictions for FIFA matches 1900-1998 with Homebonus for different $v / t$

We see WPD Normal makes in general the best predictions for FIFA matches. The best model during the WC is WPD Equality. It is difficult to decide which $v / t$ is the best choice and which model we want to choose to tune the parameters. We decided for $v / t=1.8$. This $v / t$ makes $61.57 \%$ correct predictions with WPD Normal (third highest value) and $61.18 \%$ with WPD Equality for FIFA matches. WC matches are predicted correct in $57.89 \%$ of the cases with WPD Equality (highest rate) and in $56.99 \%$ with WPD Normal.

Now we vary $\beta$. In Figure 5.34 we see that $\beta=1.5$ seems to be a good choice because the combination of correct predictions for FIFA, WC and EC matches is there the best. We used already with the Pluspoint model a $\beta$ around 1.5.

Until now we have not changed $\gamma$. All results until now are done with the classic FIFA version 4, 3, 2.5, 1. We found already with the Pluspoint model that the predictions are better when friendly and qualifier matches are rated with the same value. So we tested this model with the match importance weights 4, 3, $2.5,2.5$. We make correct predictions during WC in $58.62 \%$ with WPD Equality and in $58.44 \%$ with WPD Normal. The prediction rate for all FIFA matches are with WPD Equality in $61.06 \%$ and with WPD Normal $61.72 \%$ of the matches correct.


Figure 5.34: Correct predictions for FIFA matches 1900-1998 with Homebonus for different $\beta$

## Evaluation

In the previous chapter we tuned our parameters with the information of Bundesliga matches between 1976 and 2014 and FIFA matches until 1998. We want now look how good our predictions with the chosen parameters are when we predict matches of Ligue 1, Premier League, Primera Division and Serie A and the FIFA matches after 1998.

### 6.1 League Matches

### 6.1.1 Homebonus

We made the best predictions with the PRD Equality Homebonus and WPD Normal Homebonus model. The chosen paramters are $d=0.9$ for the random surfer parameter and $\lambda=2.5$ for the time span. In the PRD Equality model is $v / t=1.82$. In the WPD Normal we have set $v / t=2.51$.

The prediction rates for all leagues are in Figure 6.1 and Figure 6.2.
We make better predictions in all Leagues except Ligue 1. One reason could be that in the French League happen more ties during a season (home wins: $45.38 \%$, ties: $29.35 \%$, away wins: $25.26 \%$ ). The tie rate in the German League is $26.30 \%$ (HW $49.30 \%$, AW $24.40 \%$ ). Because we only bet on wins, all matches that end with a tie are wrong predictions. The best result was in the Premier League. But even there is the tie rate higher than in the Bundesliga with $25.47 \%$ (HW $46.95 \%$, AW 27.58\%).
A reason why the WPD makes better predictions than the PRD is that Bundesliga consists of 18 teams. But all other considered Leagues have 20 teams. When more teams are in the ranking the average PageRank per team will consequently be smaller. That is a problem for the PRD models if we adjust the bonus $\rho$ with matches from the Bundesliga.


Figure 6.1: Correct predictions for different leagues with PRD Equality Homebonus


Figure 6.2: Correct predictions for different leagues with WPD Normal Homebonus

### 6.1.2 Pluspoint

Our best parameters $\lambda=2.5, v / t=3.06$ and $d=0.9$ with WPD Normal Pluspoint predict $51.88 \%$ Bundesliga matches correct between 1975 and 2014. We forecast $48.02 \%$ Ligue 1, $53.52 \%$ Premier League, $52.75 \%$ Primera Division and $51.45 \%$ Serie A matches correct with those parameters. Again, Ligue 1 is more
than $3 \%$-points worse than the correct prediction rates of the other 4 leagues.


Figure 6.3: Correct predictions for different leagues with WPD Normal Pluspoint
The prediction rates with the Pluspoint model are similar to the rates with Homebonus. Some leagues work slightly better or equal like Primera Division or Ligue 1, others make little bit less predictions correct like the Premier League.

### 6.1.3 Neutral

When we test our parameters for PRD Normal and PRD Equality we get the results in Figure 6.4 and 6.5. The chosen parameters are written in the plot figures. With PRD Normal, we predict $46.36 \%$ Ligue 1, $43.69 \%$ Premier League, $44.24 \%$ Primera Division and $49.81 \%$ Serie A correct. $46.32 \%$ Ligue 1, $44.31 \%$ Premier League, $44.58 \%$ Primera Division and $50.31 \%$ Serie A matches are correctly with PRD Equality.
The best correct predictions with Neutral are done for the Italian league. Almost or even more than $50 \%$ of the matches are predicted correctly. The same behaviour shows the predictions for Ligue 1 . We make there only slightly worse predictions than when we would respect the home advantage.

We conclude that we make better predictions for all leagues when we respect the home advantage.

### 6.1.4 Combo

In Figure 6.6 are the results for the PRD Equality Combo model. Ligue 1 makes the worst predictions. We make this time the best predictions with Primera


Figure 6.4: Correct predictions for different Leagues with PRD Normal Neutral


Figure 6.5: Correct predictions for different leagues with PRD Equality Neutral

Division and not with Premier League matches. When we compare the results of Combo, Pluspoint and Homebonus, the Pluspoint and Homebonus work slightly better than the combination model. We conclude that we can not improve our predictions when we use the combination.


Figure 6.6: Correct predictions for different leagues with PRD Equality Combo

### 6.1.5 TwoTeams Neutral

The TwoTeams Neutral model is a real improvement to the simple Neutral model. The correct predictions are almost as high as with the models which give different points for home and away results or add a small bonus to the home team.


Figure 6.7: Correct predictions for different leagues with PRD Equality TwoTeams Neutral

PRD Equality and WPD Normal made almost the same correct prediction rate for the Bundesliga. But when we use TwoTeams Neutral to predict matches of the other leagues, WPD Normal works better than PRD Equality.


Figure 6.8: Correct predictions for different leagues with WPD Normal TwoTeams Neutral

### 6.1.6 TwoTeams Pluspoint

In Figure 6.9 we tried the chosen parameters on the Bundesliga matches. When we compare the prediction rates of the TwoTeams Pluspoint and the TwoTeams Neutral model, we see that we can improve the correct predictions with the combination of TwoTeams and Pluspoint. But the results in Figure 6.9 are worse than the prediction rates of the single Pluspoint model.

### 6.1.7 Conlusions for Leagues

We have now tried six different prediction models. For the Bundesliga we made the best predictions ( $52.09 \%$ ) with PRD Equality Combo. The correct predictions of the French league were never higher than $50 \%$. The best value with $48.02 \%$ was done with WPD Normal Pluspoint. The predictions with the Premier League were with almost all prediction models good except the Neutral model where we do not consider the home advantage. The highest correct prediction rate is done with WPD Normal Pluspoint ( $53.52 \%$ ). The correct predictions for the Primera Division were always similar to the prediction results of the English league. The best predictions, $52.75 \%$ of the matches, are made with WPD Normal Pluspoint


Figure 6.9: Correct predictions for different leagues with WPD Normal TwoTeams Pluspoint
and WPD Normal Homebonus. The highest prediction rate for Serie A was $51.72 \%$ with WPD Normal Pluspoint.

The results show clear that more matches are predicted correct if we respect the home advantage. The best three models are Homebonus, Pluspoint and Combo which make almost the same good predictions.

The TwoTeams model works already good in combination with Neutral. We can not make significant improvements when we combine Two Teams with Pluspoint.

We implemented two different prediction methods to see how good our results for the leagues are. The first calculates for each team a point average. This point average are all won points divided by the number of matches in the time span $\lambda=2.5$ years. With won points are meant the points during the usual league season. A team earns there three points for a victory, one for a tie and zero for a loss. The second prediction model looks what the result of the last match (the leg game) between the two teams was and predicts exact the same result. Figure 6.10 and Figure 6.11 show that our correct prediction rates are clear higher than the rates in those models.

We see that both models, the points and leg game, make worse predictions than we could make with FootballRank.


Figure 6.10: Correct predictions for different leagues with point prediction model


Figure 6.11: Correct predictions for different leagues with the leg game prediction model

### 6.2 FIFA matches

We want now see how our parameters work on the FIFA matches between the 1st January 1999 and 11th June 2014.

### 6.2.1 Pluspoint

We can see in Figure 6.12 that we predict $61.19 \%$ of the matches correct. Stunning is that the prediction rate for the European Championships is $58.06 \%$. For the other models it was always difficult to have high prediction rates for EC matches and all FIFA matches as well.


Figure 6.12: Correct predictions for FIFA matches between 01.01.1999 and 11.06.2014 with Pluspoint WPD Normal

### 6.2.2 Homebonus

We predicted with WPD Normal $61.72 \%$ of all FIFA matches correct and $58.44 \%$ of the World Cup matches for the time before 1999. WPD Equality predicted $61.06 \%$ of the FIFA matches and $58.62 \%$ of the World Cup matches correct. We test now our best parameter combination on the matches between 1999 and 2014.

We predict $61.51 \%$ and $61.27 \%$ of the matches correct with our best two models WPD Normal and WPD Equality. During the WC we have a correct prediction rate of $59.38 \%$ and $58.85 \%$. The results are quite similar to the predictions before 1999. The biggest change is the prediction rate for the World Cup with WPD Normal. We raised could raise the rate from $58.44 \%$ to $59.38 \%$.

The best predictions for the international matches made Homebonus WPD Normal. We want to show here the ranking on the day before the World Cup started, 11th June 2014, calculated with this model. The table for the best 20 teams is

FIFA 01.01.1999-11.06.2014 - Homebonus WPD Normal ( $N=10, d=0.9, \lambda=8, \beta=1.5, v / t=1.80, \sigma=0.42, \gamma=[4,3,2.5,2.5])$


Figure 6.13: Correct predictions for FIFA matches between 01.01.1999 and 11.06.2014 with Homebonus WPD Normal


Figure 6.14: Correct predictions for FIFA matches between 01.01.1999 and 11.06.2014 with Homebonus WPD Equality
shown in Table 6.1.

Table 6.1: FootballRank table on 11th June 2014

| Rank | Team | PageRank |
| :---: | :---: | :---: |
| 1 | Brazil | 0.02810 |
| 2 | Spain | 0.02412 |
| 3 | Argentina | 0.02197 |
| 4 | Germany | 0.01965 |
| 5 | Italy | 0.01733 |
| 6 | Mexico | 0.01709 |
| 7 | France | 0.01665 |
| 8 | Netherlands | 0.01607 |
| 9 | USA | 0.01602 |
| 10 | Chile | 0.01598 |
| 11 | Uruguay | 0.01590 |
| 12 | England | 0.01562 |
| 13 | Portugal | 0.01397 |
| 14 | Paraguay | 0.01358 |
| 15 | Colombia | 0.01322 |
| 16 | Ecuador | 0.01271 |
| 17 | Japan | 0.01258 |
| 18 | Switzerland | 0.01181 |
| 19 | Venezuela | 0.01093 |
| 20 | Korea Rep | 0.01026 |

### 6.2.3 Conclusion of FIFA matches

Our best ranking, WPD Normal Homebonus, predicts $59.375 \%$ of the matches on the last three World Cups correct. We can compare it with the predictions of the FIFA ranking. We assume that in a match between team $i$ and $j$, we predict a victory of team $i$ if team $i$ is higher ranked as team $j$. FIFA makes then $53.65 \%$ correct predictions. Therefore, FootballRank makes better predictions than the FIFA ranking for World Cups. The aim of this thesis was to create a better ranking for the national teams than the FIFA has. We can say, that we accomplished this goal.

Our results have shown for the FIFA matches (and the leagues) that the predictions are better when we respect the home advantage. We had almost the same results for the Pluspoint and Homebonus model. The Neutral model worked always worse. The FIFA ranking[?] and the ELO ranking [3] do not respect any factor for home/away advantage. Only the SPI ranking takes the home advantage into account.

We had three different variants to create the graph with the matches. The conservative variant, Normal, worked the best. Equality was always not so much worse but in the end, Normal made the best predictions. Equality modifies the edge weights and adds a self-link if there are unequal weights for points on both edge directions. This did not made a real improvement. Perhaps, the problem is that in FIFA matches do teams very rarely play more than one or two matches against each other. The edge modification works only if there are weights on both edges. There are probably often not on both edges weights. In the league matches did Equality models made the best predictions in combination with TwoTeams Neutral, Neutral and Combo. But there are more matches in the graph and Equality works probably better.

A bad thing of FootballRank is that the PageRank of teams can grow or drop even if they have not played a match. This happens when former opponents play matches. So, an old victory of one year ago can still be the reason why the PageRank of a team can grow. It is difficult to say if a proper ranking is allowed to have this characteristic. The good thing of that behavior is that we do not have to set a factor for the strength of the opposing team. The strength is difficult to identify. Some rankings do it with different rank positions, other do it with the points in the ranking. But the best way is if the ranking does it by itself.

The ranking will be more reliable if teams play against a big number of various opposing teams. In the FIFA matches play teams only during World Cup, Confederations Cup and in friendlies against teams of different confederations. The other matches are all against teams of the same region. Probably consists our graph of six parts (because of the six confederations). The nodes inside those parts are well connected. But the different parts have only less edges between them. This could also be the reason why we predict more matches correct when we increase the $\gamma(4)$ for friendlies. Edges, which are created with friendlies, have then a bigger edge weight.

## Outlook

An improvement for our ranking could have been if we would have chosen the bonus $\rho$ dependent on the teams in the graph. That is perhaps the reason why we can make good predictions in FIFA matches with PRD models and with WPD models which use $\sigma$. Section 6.1 .1 shows that we can make almost the same good predictions with WPD Homebonus and PRD Homebonus for the league seasons. But there are around 20 teams in the graph. In the FIFA matches are right after 1900 only a few teams but now in 2014 are around 200 nations.

When we have a look on the tie rate during the World Cups between 2002 and 2010. Only $20.31 \%$ ended in a tie. $23.27 \%$ of all FIFA matches were tied. Less ties are always good for our ranking. The FootballRank would perhaps work better in sports where ties do not exist, like ice hockey or tennis.

Another thing is the relation of $v / t$. We always changed the relation of $v$ and $t$. For the league seasons is that okay, because it is the only factor for the edge weight. But perhaps we could have improved the ranking when we vary $v$ and $t$ independently for the FIFA matches.

There is another idea to create the graph. We only used the results (win, tie or loss) to create the edges and the edge weights. The SPI, explained in Section 2.2, gives each team two values for their offensive and defensive score. We could create two graphs. In one graph are the edges defined with the scored goals, the other graph has edges determined with the allowed goals. Each team would have two PageRanks. One for the scored and one for the allowed goals. Before a match, those two values of both teams could be compared to make a prediction.

Another way to determine the weights of the edges is with the help of the score difference. In our graph it does not matter how high a win is. We do not take the score difference into account. The ELO ranking respects the goal difference[3]. A ranking which was made for the National Football League (NFL) in the United States creates the edge weights with the scoring difference too[10]. The direction of the edge goes from the looser team to the winner team and the edge weight
is defined by the positive score difference.
Of course the best ranking could be done, if each team plays against all other teams. Everybody would have the same difficult matches. If we have $n$ teams we would need $n$ ! matches. But it is impossible to arrange all those matches inside a reasonable time span.

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