Collusion Resistance in Peer Grading

Semester Thesis

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Abstract

In this work, we analyse how to make peer grading resistant against a group of colluding students. We define "colluding students" as a group of students who want to improve the score of others within that group (e.g. friends). For this reason we studied different scenarios with different behaviours of both honest and colluding students. We analysed several grading schemes and their behaviours in different scenarios. We found that the colluding resistance can be maximized with an optimized ordering of the students and could be reduced to a graph theory problem. We showed that for a given scenario with correct honest graders and our provided ordering the best grading scheme out of the ones presented is the median while in the second scenario with honest students being correct with their average it is taking the average.
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Chapter 1

Introduction

As one could see in this very special last year, due to the COVID-19 pandemic, online learning is becoming more and more important. Not only in university but also in elementary, grammar or vocational schools, classes needed to be held online. A lot of them will continue to do so, given the advantages of online classes. All you need for such a class as a student is access to the internet and a place to work. Students can work everywhere and every time in their own tempo. That is why, even before the pandemic, massive open online courses (MOOCs) have become more and more popular with courses distributed to everyone.

Although these courses have a lot of advantages, it is not so easy for exams. Of course most of them (whenever possible) are done with multiple choice questions and graded in automated manner. Where this is not possible (e.g. essays) there is a lot of time-consuming work needed to be done by the grading experts. This is why such works are graded with Peer grading schemes. This saves a lot of work for the teachers, saves time for the students (i.e. they get faster feedback of their work) and as shown in some studies [1, 2], Peer grading helps the students to improve their own understanding of the given topic.

Peer grading has a lot of advantages. The challenge is to incentivize the students to grade truthfully and correctly since this is hard work. Even if they do so, it is still possible that a group of friends try to improve another-one’s grade. This constitute the topic of the thesis: a group of students tries to collude. We try to minimize the score improvements in such cases.

In chapter 2, one can find a closer look of the setting we consider in this work and the notation used. Chapter 3 contains the first scenario with the honest students grading correctly and a discussion about the used grading scheme with two different assignments of the students. In chapter 4 we have a look at a more realistic scenario where honest graders might not grade correctly. It also contains a discussion about possible grading schemes in this setting, and the transfer into a graph-theory problem. Finally chapter 5 concludes what was done.
1. Introduction

1.1 Related Work

A lot of literature on Peer assessment is about course evaluation, the pedagogical aspects or about selections and evaluation on scientific work.

Overall, Peer assessment can be partitioned into three different groups: selection, ranking and grading. Selections, as in [3], are used to pick the best works (e.g. k out of a set with n works) without offering any additional information. Solutions to ranking, where the works should be ranked from best to worst without providing grades, are provided in [4, 5]. However [4] shows that ranking is not liked as much as grading, where works are graded with a grade, by the students. Additional grading is the most powerful Peer evaluation since out of given grades one can easily generate a ranking and also do a selection.

There are several approaches on how to develop tools for Peer grading as e.g. in [6, 7]. Especially Toby Walsh’s not monotonic PeerRank Method is achieving remarkable results and therefore have been reviewed by [8, 9]. There even changes have been suggested and tested. There are also a few papers and work about strategyproofnes [3, 10, 11, 12] but all of them about selection. Strategyproofnes means that students can not benefit by reporting insincere valuations. [4] provide a game-theoretic approach to incentivize students to grade correctly.

Although the therm "collusion-resistance" is mentioned in some works, to the best of our knowledge, it has not been deeply studied yet.
In our setting, we consider a course with a lot of students. They all need to be graded with Peer grading. We want to give the students already with our global setting an incentive for grading correctly and truthfully. Doing so is hard and time-consuming work, hence we do not want every student to grade every other student. It has been showed in [13] that the incentive for grading truthfully is much higher if there is less work to do.

We suppose there are \( n \) students who need to be graded. Every student receives \( m \) grades and grades \( m \) other students without grading their own work. The grades are normalised to the interval \([0,1]\). The final score of a student \( i \) is denoted by \( \text{score}(i) \). There are \( c \) students, who are trying to collude, what means that they try to improve the score of other colluding students and if possible even the sum of their scores.

**Definition 2.1** (colluding group). \( c \) students form a colluding group, if they try to improve the score of another student of the colluding group or even the sum of their scores \( \sum_{i \in C} \text{score}(i) \).

### 2.1 Meanings and Definitions

In this section we explain a few often used terms.

1. \( \text{grade}_j(i) \): the grade given by student \( j \) to student \( i \).
2. \( \text{score}(i) \): the final grade of student \( i \).
3. \( \text{GS} \): the applied grading scheme.
4. \( \text{CS} \): the strategy applied by the colluding group.
5. \( \text{GT}(i) \): the ground truth for the score of student \( i \).
6. \( \text{HG} \): a sub-group of honest graders

**Definition 2.2** (symmetry of a grading scheme). If all given grades are "flipped", then the score is also "flipped". Flipping a grade or a score means that \( \text{grade}_j(i) \)
flipped is \(1 - \text{grade}_i\), e.g. let’s assume that a student i receives the grades \(g_1, g_2, ..., g_m\) and that the score is \(s\). If the student now receive the grades \(1 - g_1, 1 - g_2, ..., 1 - g_m\), the score is \(1 - s\).

2.2 Collusion Error

For measuring the influence of a colluding group, we define two types of collusion-error coefficients. One is for an individual student and the other one for a group depending on the ground truth (GT), the grading scheme (GS) and the colluding strategy (CS). The error coefficients consider the worst-case scenario over all colluding strategies.

**Definition 2.3 (Individual Collusion-Error Coefficient).** The individual collusion-error coefficient measures the distance between the score \(\text{score}(i)\) of student \(i\) and the ground truth.

\[
E_{\text{id}}(GS) = \max_{i \in C, CS} \left| \text{score}(i) - GT(i) \right|
\]

**Definition 2.4 (Group Collusion-Error Coefficient).** The group collusion-error coefficient measures the sum of distances between the score \(\text{score}(i)\) of student \(i\) and the ground truth for each student in a group.

\[
E_{\text{group}}^C(GS) = \max_{CS} \left( \frac{1}{|C|} \sum_{i \in C} |\text{score}(i) - GT(i)| \right)
\]
Chapter 3

Correct Honest Graders

In this chapter we start with a simple scenario. We assume that the honest students are good graders and therefore grade correctly with an error of $\pm \delta$ ($\delta > 0$). Further we consider two different scenarios, which are described in the respective sections below.

3.1 Grading Scheme

We can partition the grading scheme into three parts. The initialization phase, the grading phase and the evaluation phase. In the initialization phase the grading graph is built up and the students are distributed to the nodes. In the grading phase the students are doing their grading and in the evaluation phase the grading scheme is applied.

Definition 3.1 (grading graph). In a grading graph the students are represented by the nodes and the grades given by the students are represented by the edges of the graph.

Initialization Phase

We want to take an ordering similar to the one mentioned in [3]. The goal is to split the $n$ students into $m$ groups and creating a graph with a long shortest circle. We therefore:

1. assign a unique id from 0 to $n - 1$ to each student
2. assign the student $i$ to group $i \mod \lfloor \frac{n}{m} \rfloor$

Since the last group does not contain $m$ students when $\frac{n}{m} \notin \mathbb{N}$, there is a problem with the second last, the last and the first grading-group.

There are $k = n \mod m < m$ students in the last grading group. Each of them grades all students in the first group and receives grades from the second last group. Because of that the students in group 0 only receive $k$ grades and the students in the second last group only grades $k$ others. For solving this problem
we build a \((m - k)\)-regular bipartite sub-graph with students-groups \(\lfloor \frac{n}{m} \rfloor - 2\) and 0 as nodes and use a circular approach where the \(i\)-th student of group \(\lfloor \frac{n}{m} \rfloor - 2\) grades students \(i \mod m, i + 1 \mod m, ..., i + m - k - 1 \mod m\) as in 3.1.

**Algorithm 1** Build \((m - k)\)-regular bipartite sub-graph

1: for \(i=0\) to \(m-1\) do
2:   for \(l=i\) to \(l+m-k-1\) do
3:     add edge from node \(i\) in \(A\) to node \(l \mod m\) in \(B\)
4:   end for
5: end for

Algorithm 3.1 builds a \((m - k)\)-regular bipartite graph.

**Proof.** Let the left partition \(A\) of the sub-graph be group \(\lfloor \frac{n}{m} \rfloor - 2\) and the right partition \(B\) group 0. The graph is bipartite since there are no edges between the vertices of \(A\) or between vertices of \(B\). Every vertex \(i\) in \(A\) has out-degree \(m - k\) by construction and every vertex \(j\) in \(B\) has in-degree \(m - k\) due to the following argument. We have an edge\((i, j)\) if \(j = i + s \mod m\) for \(s \in \{0, ..., m - k - 1\}\), which means that we have an edge\((i, j)\) for \(j = i - s \mod m\) for \(s \in \{0, ..., m - k - 1\}\). □

**Theorem 3.2.** Our assignment guarantees that every student grades \(m\) others and receive \(m\) grades.

**Proof.** The grading-groups 1 to \(\lfloor \frac{n}{m} \rfloor - 1\) all have in-degree \(m\) by construction and the grading-groups 0 to \(\lfloor \frac{n}{m} \rfloor - 3\) and \(\lfloor \frac{n}{m} \rfloor - 1\) all have out-degree \(m\) by construction. The grading-group 0 has in-degree \(m - k\) and the group \(\lfloor \frac{n}{m} \rfloor - 2\) out degree \(m - k\). Adding the bipartite graph additionally the students in these groups grade and are graded by \(k + (m - k)\) students. □

**Grading Phase**

Every student from group \(i \mod \lfloor \frac{n}{m} \rfloor\) is grading all the students from group \(i + 1 \mod \frac{n}{m}\).

**Evaluation Phase**

It seems clear that there are cases, where it is impossible to get a score of a student close to the ground truth. This is the case, when \(\frac{n}{m}\) or more grades of a student are given by the colluding group. In this case, especially when there are only two given grades (i.e. one by the colluding group and one by the honest graders), it is not possible to decide which group is right. With this in mind we only pay attention to all other cases where there are more grades given by
3. Correct Honest Graders

Figure 3.1: example for the last two and the first grading-groups with $m = 5$ and $n = 23$

honest graders than by the colluding group to one student. Since the honest graders in this scenario are grading correctly, the best grading scheme is to take the median for the final score. That way we will always end up either with a honest grade or a grade from a colluding student, which in return has to be in the interval $\pm \delta$ of the ground truth.

3.2 Deterministic Case

In this first scenario we assume a worst-case-scenario, were the assignment is pre-known by the students (e.g. the unique ids or based on the alphabetical order). If this is the case, we still can get good results as long as the colluding group is not too big.

Lemma 3.3. If there are $< \frac{m}{2}$ colluding students grading a student, the error of this student is at most $\delta$.

Proof. Since $c \leq \frac{m}{2}$ a colluding student receives at most $\frac{m}{2}$ grades from other colluding students which therefore can only shift the median by $\frac{m}{2}$ positions. So the final grade is always in the range of the honest graders (GT $\pm \delta$).

Theorem 3.4 (Deterministic Case). For $c \leq \frac{m}{2}$ $E_{|c|} \leq \delta$ and $E_{|c|}^{\text{group}} \leq |c| \cdot \delta$.

Proof. Since there are $c \leq \frac{m}{2}$ colluding students in the worst case one of them is graded by the other $\frac{m}{2} - 1$. Then the result follows from lemma 3.3.
Note that the score is in the interval of the honest graders also in the worst-case.

### 3.3 Probabilistic Case

In the second scenario, we consider a much more realistic scenario, where the assignment is not pre-known and in a random order. Before we carry on we need to define the failing probability.

**Definition 3.5 (failing probability).** The probability of our scheme to "fail" is $P\{\text{fail}\}$. Failing means, that there is at least one grading-group with $\geq \frac{m}{2}$ colluding students. $P\{\text{fail}\} = P[\exists \text{ group with } \geq \frac{m}{2} \text{ colluding students}].$

**Theorem 3.6 (Probabilistic Case).** For $c < \frac{n-m}{2}$; $E_{|c|} \leq \delta$ and $E_{|\mathcal{C}|} \leq |\mathcal{C}| \delta$ with a failing probability of $P\{\text{fail}\} < \frac{n}{m} \times e^{-0.003m}$ for $\geq \frac{m}{2}$ colluding students in one grading-group.

**Proof.** $P\{\text{fail}\} = P[\exists \text{ group with } \geq \frac{m}{2} \text{ colluding students}] \leq \sum_{i} P[\text{group } i \text{ has } \geq \frac{m}{2} \text{ colluding students}] \leq \frac{n}{m} P[\text{group } 0 \text{ has } \geq \frac{m}{2} \text{ colluding students}].$

Therefore we have a look at the draw of group 0 and analyze the probability that this group has at least $\frac{m}{2}$ colluding students. We use Chernoff [14] to provide an upper bound for $P[\text{group } 0 \text{ has } \geq \frac{m}{2} \text{ colluding students}].$

We define $m$ independent Bernoulli random variables $X_i$ such that $P[X_i = 1] = \frac{c}{n-m+1}.$

Let $C_i$ denote the event that the $i$-th student of the group colludes.

- $P[C_1] \leq \frac{c}{n} \leq P[X_1]$
- $P[C_2] \leq \frac{c}{n-1} \leq P[X_2]$
- ...
- $P[C_m] \leq \frac{c}{n-m+1} \leq P[X_m]$

$C = \sum_i C_i$ denotes the number of colluding students in group 0 and $X = \sum_i X_i.$

Note that $P[\text{group } 0 \text{ has } \geq \frac{m}{2} \text{ colluding students}] = P[C \geq \frac{m}{2}] \leq P[X \geq \frac{m}{2}].$

Using Chernoff [14], we obtain an upper bound for $P[\text{fail}].$ We get $\leq Pr[X \geq 0.5 \times m > (1 + 0.15) \times 0.43 \times m] \leq e^{-0.15^2 \times 0.43m} < e^{-0.003m}.$ Therefore $P[\text{fail}] \leq \frac{n}{m} \times P[\text{group } 0 \text{ has } \geq \frac{m}{2} \text{ colluding students}] \leq \frac{n}{m} \times e^{-0.003m}$

If our scheme is not failing, the results from the deterministic case, as in 3.4, applies.

□
Obviously it is not a very realistic case that all the honest students also grades correctly. Due to not everybody being able to grade correctly, as in \cite{6}\cite{15}, we have to use other assumptions. Therefore we consider a more realistic scenario with a few assumption close to normal distribution.

I - The average of the honest graders (HG) grading a student is the ground truth (GT):

\[
\frac{1}{|HG|} \sum_{j \in HG} grade_j(i) = GT(i)
\]

II - The average of the distance from the ground truth is equal to a number \(\sigma > 0\):

\[
\frac{1}{|HG|} \sum_{j \in HG} |grade_j(i) - GT(i)| = \sigma
\]

Additionally in this chapter, we discuss the different collusion errors (i.e. individual and group).

### 4.1 Grading Scheme for Individual Collusion Error

In this section we only consider one student graded by \(m\) other students. The set of graders is denoted by \(G\). For finding the optimal grading scheme for an individual, we need the following definitions.

I - The average of all grades (including those from the colluding group) received from a student \(i\) is \(avg(i)_{\text{moved}}\):

\[
\frac{1}{m} \sum_{j \in G} grade_j(i) = avg(i)_{\text{moved}}
\]
II - The average of the distance between a given grade and the ground truth of all given grades (including those from the colluding group) received from a student is equal to a number $\sigma(i)_{moved} > 0$:

$$\frac{1}{m} \sum_{j \in |G|} |\text{grade}_j(i) - \text{avg}(i)_{moved}| = \sigma(i)_{moved}$$

In the following subsections we show a few grading scheme approaches.

### 4.1.1 The $\sigma$-approach

We assume $\sigma$ is known. The first idea is to take the distribution and iterate with the known $2 \cdot \sigma(i)$ until we find the interval with the most grades, which we then will only take into account for the final grade. For the final score we take the average of the grades given in this interval.

This first approach has a few caveats because $\sigma(i)$ is not too big and in practice, not known. In a case where all colluding students give the same grade, which is clearly possible, we will likely end up with choosing an interval including the colluding group. We do not want this because this way we would end up with a final score close to or exactly the grade given by the colluding group. In a worst case scenario we would have $E_{|G|} = 1$

![Figure 4.1: The first grading-scheme-approach (colluding-group in black)](image)

### 4.1.2 The $\sigma_{moved}$-approach

Since $\sigma(i)$ is not known, we can just take the given grades and compute the average distance $\sigma(i)_{moved}$ and then use the same idea as in the first approach.
This approach works well for the most extreme cases, for example, if GT is close to 0 and the colluding group all giving grades around 1, we would obtain $E_{C|} = 0$. However, if GT is around 0.5 and the true $\sigma(i)$ is high, we would end up again with an interval including the colluding group and the same result as in the first approach.

![Figure 4.2: The second grading-scheme-approach](image)

### 4.1.3 The delete-outside-$\sigma_{moved}$-approach

The third scheme is to take $\text{avg}(i)_{\text{moved}}$ and remove all the grades which are more than $\sigma(i)_{\text{moved}}$ far from $\text{avg}(i)_{\text{moved}}$. For the final grade, we take a new average from the remaining grades.

The third approach is overall the best grading scheme out of the three options presented here. It works well for most of the cases. There are cases, where also this scheme fails and would end up with the final score equal to the colluding grades. This happens when a grader is honest but grades very badly, e.g. gives the same grade as the colluding group. If this happen, a bad honest grader might end up "helping" the colluding group. Therefore, the final score would be based on the grades of the colluding group.

### 4.1.4 Taking the median

With taking the median we have the same problem as with the third approach. If a student grades honest but very badly and ends up with giving the same grade as the colluding group, we end up with a colluding grade.
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4.1.5 Average

By using the average as a grading scheme the score(i) of student i is equal to \( \text{avg}(i)_{\text{moved}} \). For this grading scheme we first need to analyse how much the average can be moved by the colluding group.

**Theorem 4.1.** The average can be moved by a maximum of \( \frac{c}{m} \times (1 - GT) \) by the colluding group.

**Proof.** Since the colluding group c is a fraction of m and tries to improve someone's score by definition of the average it can be moved by \( \frac{c}{m} \times (1 - GT) \). \( \square \)

**Theorem 4.2 (Average Graders).** If the grading scheme is symmetric, \( E_{ij} \) is minimized for score(i) = \( \text{avg}(i)_{\text{moved}} \).

**Proof.** To prove this statement, we provide two scenarios given a symmetric grading scheme (excluding the average) is applied. In the first scenario, we assume that the work that the students have to grade is low-grade. A set of honest graders is very inaccurate, giving the same grade as the colluding group. We use CG to denote the set containing the colluding students and these inaccurate honest students, and HG to denote the set containing the remaining honest students. In this scenario, the students in HG give the same grade. We assume that \( |CG| = |HG| \).

Let \( \text{grade}_{leHG}(i) < \text{grade}_{leCG}(i) \). For an symmetric grading scheme being better than the average \( |\text{score}(i) - \text{grade}_{leHG}(i)| < |\text{score}(i) - \text{grade}_{leCG}(i)| \) would be the result.

In the second scenario, the work the students are grading is good, and none of these students are part of the colluding group. Half of these students (denoted by \( HG_1 \)) give the same grade as \( HG \) in the first scenario, while the other half...
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(denoted by $HG_2$) is inaccurate and gives the same grade as $CG$ in the first scenario.

In the second scenario, we would expect the final score to be closer to the grade given by $HG_1$. However, the inputs for the grading scheme are exactly the same as in the first scenario.

![Figure 4.4: worst-case scenario for symmetric grading schemes](image)

4.2 Grading Scheme for Group Collusion Error

Since there is no way to prevent students to improve another student in our setting, we try to find a possibility that the students can not improve their group-score by a significant amount. We assume that the students know the structure of the graph and can choose the nodes, which will help to understand the worst-case scenarios.

We try to find a grading-graph that already makes it as hard as possible to improve many grades within the colluding group. The graph has to fulfil the following conditions:

I - the graph needs to be $2m$ regular, since every student grades $m$ others and receives $m$ grades.

II - the number of edges in the sub-graph defined by the colluding group should be minimized

III - there should be no pair of students grading each other.
IV - we want the graph to be sparse intuitively and therefore it should be connected.

4.2.1 Graph

For these conditions we first came up with the graph in figure 4.5:

Algorithm 2 Build Graph

1: for $i=0$ to $n-1$ do
2:    for $k=0$ to $m-1$ do
3:        add edge from node $i$ to $i+1+k\cdot\lfloor n/m \rfloor$
4:    end for
5:  end for

Figure 4.5: graph with $n=12$ and $m=2$

The graph we came up with has the following characteristics:

I - none of the grading schemes presented in this chapter is optimal for this graph. It seemed likely that taking the median would be the best grading scheme. There are cases, e.g. if there are less than $\frac{m}{2}$ graders of someone from the colluding group, where we would end up with a honest grade as the
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median. But on the other hand it is enough to have \( \frac{m}{2} + 1 \) grades given from
the colluding group to improve someones score by the maximum. At this point
taking the average for the score would be much better, since there the score can
only improved by \( \frac{c}{m} \ast (1 - GT) \).

II - The graph is connected. If it would not be so, the colluding group would
take the smallest connected component and in the worst case end up in a group
grading each other without other (honest) graders.

III - The graph does not allow students grading each other.

IV - There are numbers for \( n \) and \( m \) (e.g. \( n = 1000 \) and \( m = 50 \)) where we
end up with a graph similar to the one presented in chapter 3. This allows the
colluding group to grade the same students and therefore the sub-graph of only
the colluding group will have a lot of edges inside.

4.2.2 Moore graph and Further Studies

Since the previous graph is not optimal we looked for other graphs that max-
imize the shortest cycle-length. Such regular graphs are Moore graphs with a
maximum number of vertices for given degree and diameter as well as a min-
uminum number of vertices for given degree and girth [16][17]. An example for
such a graph is the Hoffman-Singleton graph as in 4.6. These Graphs do not exist

![Figure 4.6: The Hoffman-Singleton graph [18]](image)

for every number of vertices. That is why further studies would need to show
how to generalize them and also to prove why they are optimal for colluding
resistance.
In this work we analysed two scenarios with different behaviours of both, the honest and colluding students. In the first scenario with correct honest graders we found, that the best grading scheme out of the ones presented is the median. In the second scenario with the honest students being correct with their average, we showed that for an individual student taking the average of all grades given is the best grading scheme out of the ones presented. For the colluding group we found, that the problem could be reduced to a graph theory problem, where further studies would be needed.


