Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Distributed Computing

Hedging the Risks of Liquidity Providers

Bachelor's Thesis

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Abstract

We analyzed several hedging strategies for both divergence loss and the profit and loss of liquidity provision. The strategies were evaluated using monte carlo simulations to compute the expected return, value at risk, and conditional value at risk. We then ran empirical tests using historical data to judge past real-life performance, which was also measured with previously mentioned metrics. The strategies evaluated employ different derivatives such as options and power perpetuals available in decentralized finance. We conclude that while, in theory, option-based strategies seem to fare better than other strategies employing power perpetuals, in practice, the latter strategies seem to perform better due to the scarce availability of options in decentralized finance.

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Chapter 1
Introduction

With the ever-increasing hype surrounding decentralized finance, the market has to find ways to effectively provide liquidity to push down transaction fees (also known as gas fees) and enable quicker matching of trades. Traders pay gas prices to miners to execute their transactions on the blockchain, and market dynamics regulate the fees. Such mechanisms are crucial to a properly functioning market and to bring the market to the mainstream. In decentralized finance, we have seen the advent of automated market makers (AMMs), which cut out the middle man and automate the process of liquidity provision.

An AMM, unlike a traditional market maker, is an algorithmic order-book-free method to find matching trades. In traditional finance, one typically has an order book where different agents can display their offers; a market maker would then use its own supply of the asset to match with the order book, thereby providing liquidity and making a profit on the bid-ask spread. We will present several strategies to hedge losses obtained by providing liquidity. These losses are unavoidable and occur on any price movement of one of the assets in the asset pair.

Our goal is, therefore, to hedge against large price fluctuations. We will see that we can achieve excellent results employing both strategies using options and strategies using power perpetuals. The latter seems to fare better in our empirical tests due to the lack of variety in options in decentralized finance.

1.1 Related Work

In the work of Akhilesh Khakhar and Xi Chen on delta hedging liquidity positions on automated market makers [1], we were introduced to hedging the profit and loss of providing liquidity instead of hedging impermanent loss. Additionally, the work presented a delta hedging algorithm that optimizes the purchase of options to best approximate the profit and loss curve of liquidity provision using options.

We have seen how weighted variance swaps can be used to hedge against impermanent loss in the work of Masaaki Fukasawa, Basile Maire, and Marcus

Wunsch [2]. We used the paper's results in our work and expanded on the results by running empirical tests on the theoretical strategy.

Jun Aoyagi presents optimal behavior for liquidity providers [3] in their work. The strategies presented in this work do not rely on hedging but on the optimal behavior of liquidity providers to maximize returns.

Chapter 2

Background

As we will use several concepts from decentralized finance that might not be evident to all readers, we will explain these concepts in this chapter.

2.1 Automated Market Makers (AMMs)

An automated market maker requires a liquidity pool to execute the trade automatically. This may occur in different ways; however, one of the most prevalent ways in decentralized finance is based on a constant function market maker (CFMM). In this thesis, we will focus on a subset of CFMMs called a constant product market maker or *xyk*-model, which is the basis of Uniswap [4].

2.1.1 Constant Product Market Maker (CPMM)

A CPMM is characterized by the product of the amount of asset X multiplied by the amount of asset Y, which remains constant while transacting (Eq. 2.1). Therefore, the price of asset X in terms of asset Y is given by dividing the amount of asset Y by the amount of asset X (Eq. 2.2).

$$x \cdot y = k \tag{2.1}$$

$$\frac{y}{x} = p \tag{2.2}$$

where x is the amount of asset X in the pool and y is the amount of asset Y in the pool. k is the constant factor, and p signifies the current price of asset X in terms of asset Y. Since k remains constant unless liquidity is added, it means that if a trade occurs, say an agent purchases Δx of asset X the amount of asset Y goes down by Δy based on,

$$(x + \Delta x) \cdot (y - \Delta y) = k \tag{2.3}$$

$$\Delta y = \frac{k}{x + \Delta x} \tag{2.4}$$

and the price moves to

$$p' = \frac{y - \Delta y}{x + \Delta x} \tag{2.5}$$

Fees For their service, liquidity providers are paid fees which in Uniswap V2 amount to 0.3% of each transaction amount in proportion to the amount of liquidity they provide. The fees are immediately returned to the liquidity pool. Therefore, after each transaction, the constant k changes [4].

2.2 Liquidity Provision in Decentralized Finance

A liquidity provider is someone who adds asset pairs to a liquidity pool in order to facilitate trading. Their portfolio consists of the assets provided to the pool, and their performance is then generally measured against a portfolio of simply holding the assets. In this thesis, we will also analyze the performance against holding US dollars which we call the dollar portfolio. To promote liquidity provision, liquidity providers are awarded fees for each transaction they facilitate based on their total contribution to the pool [4]. We will see in Section 2.2.1 why these fees are so important.

2.2.1 Divergence Loss

Divergence loss is the loss in value a liquidity provider endures when the price moves or diverges. It occurs because the portfolio changes when the price changes. That means that if there is an exchange from asset B to asset A we see a reduction in our amount of asset A held and an increase in asset B. With this change, the price of the asset pair fluctuates in the opposite direction of our change in portfolio, causing the loss. This loss is also known as impermanent loss due to its impermanent nature, which means if the price returns to the initial price when depositing, the loss vanishes. Divergence loss $\Delta\ell_{LP}$ is characterized by the following equation:

$$-\Delta\ell_{LP} = \frac{2\sqrt{p_r}}{1 + p_r} - 1 \tag{2.6}$$

where p_r is the ratio between the starting price p_0 and the price at time t p_t. The full derivation can be found in Appendix A.

2.2.2 Profit and Loss of Liquidity Position

For an investor, who does not hold both asset pairs to provide liquidity, the comparison above will not suffice. They want to compare their performance to not being invested in decentralized finance instead of comparing it to holding the asset pair. In [1] we were introduced to the profit and loss formula for AMMs with uniform liquidity, such as Uniswap V2, where the comparison is not between holding or providing liquidity but between the initial value of the investment and providing liquidity. The P/L curve is negative in 50% of cases and positive the other times. The following equation characterizes it:

$$-\Delta \ell_D = \sqrt{p_r} - 1 \tag{2.7}$$

As before the derivation can also be found in Appendix A.

Chapter 3
Strategies

In this chapter, we will analyze several strategies to hedge portfolios that contain liquidity positions based on the selected investment measurement. The first part will concentrate on strategies that compare the performance to holding the two asset pairs, while the second part concentrates on the performance compared to keeping cash in dollars.

In most of the literature on divergence loss, the assumption is that one either holds an equivalent amount of the two assets or invests in a liquidity position of the asset pair; hence the comparison occurs through the former measurement. However, for many investors that would not otherwise hold investments in crypto, the latter measurement is more appropriate since they can measure their performance against not holding crypto at all.

3.1 Metrics

We will analyze the strategies along multiple metrics to judge their theoretical performance. The metrics selected here are expected profit/loss, value at risk, maximum loss, and maximum profit. These metrics will give us an indication of profit and loss performance and risk exposure, two indicators that are great for comparing different strategies. We will quickly explain the individual metrics before diving into the strategies.

3.1.1 Expected Profit/Loss

To compute the expected profit and loss performance, we use the law of the unconscious statistician to evaluate our strategy based on a function that describes our strategy. Let the random variable R denote our return, and the random variable P denote the price movement. In that case, our return is defined as the strategy function s evaluated with P. Hence we have:

$$R = s(P) \tag{3.1}$$

The assumption is that the price of asset A in terms of asset B is log-normally distributed with mean 0 and variance σ^2 as shown in [5]. In particular, σ denotes the volatility of price P. Hence to compute our expected return, we apply the law of the unconscious statistician and obtain:

$$P \sim \text{log-normal}(0, \sigma^2) \tag{3.2}$$

$$\mathbb{E}[R] = \mathbb{E}[s(P)] = \int_0^\infty s(x) f_P(x)\ dx \tag{3.3}$$

In addition, in the theoretical part of the thesis, we assume that we have no income from fees and maintain a risk-free rate of 0%. In the results presented, we use daily volatility of 5%. To evaluate the expected return, we applied Monte Carlo simulation under Geometric Brownian Motion assumptions as described in [6] running 10,000 steps of 30 days each.

3.1.2 Value at Risk (VaR)

Value at Risk (VaR) measures the risk exposure of a specific strategy. It states that with probability $1-p$, the portfolio will not lose more than $x\%$ in a given time frame. VaR is computed as the $(1-p)$-quantile (inverse CDF) of the underlying returns R, which are assumed to be normally distributed; we hence have:

$$p\text{-VaR} = F_R^{-1}(1-p) \tag{3.4}$$

where F_R denotes the CDF of R.

In practice, we evaluated the VaR, like with the expected return, through Monte Carlo simulations based on Geometric Brownian Motion. In our measurements, we assume a 0.05-VaR. We will, however, omit the probability from our text and always refer to the value at risk as VaR.

3.1.3 Conditional Value at Risk (CVaR)

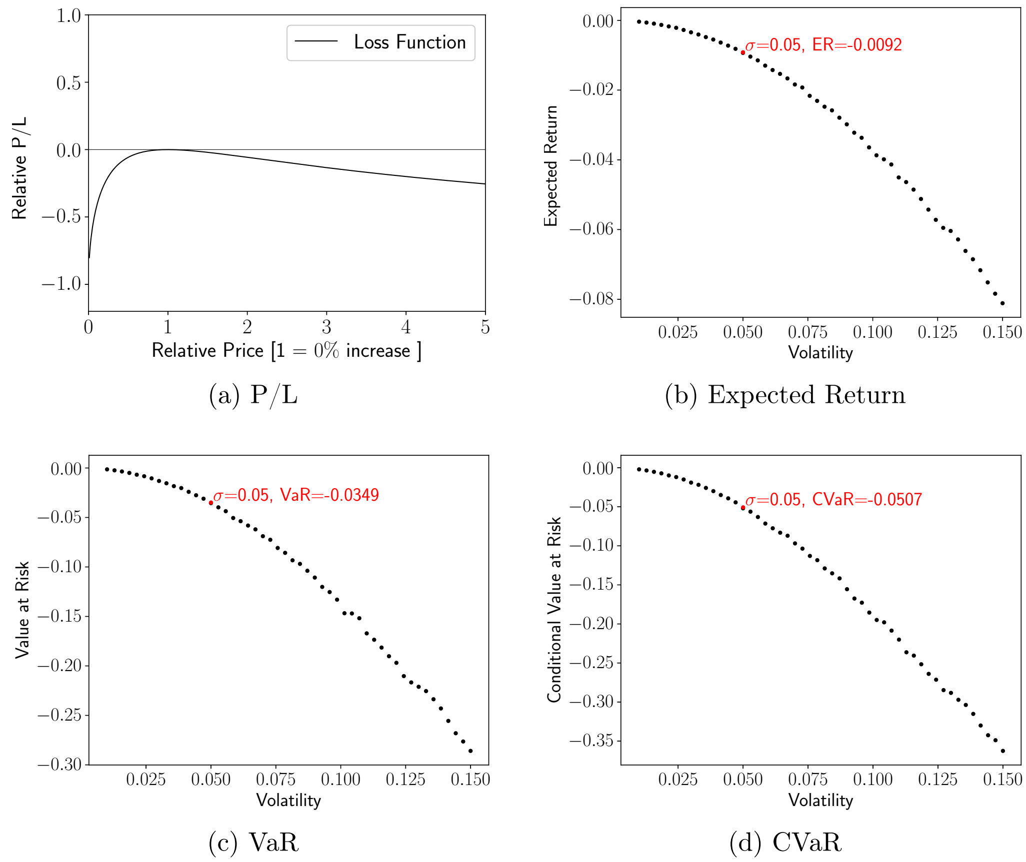
Conditional Value at Risk (CVaR) measures the mean losses in the scenarios that are less than or equal to the VaR. Then the CVaR is given by:

$$\text{CVaR} = \frac{1}{p} \int_0^p q\text{-VaR}\ dq$$

We evaluated CVaR using Monte Carlo simulations based on Geometric Brownian Motion.

3.2 Hedging Divergence Loss

To be able to hedge divergence loss, we first have to analyze the unhedged portfolio. The unhedged portfolio consists of a split of Asset A, and Asset B invested



(a) P/L (b) Expected Return

(c) VaR (d) CVaR

Figure 3.1: The metrics of unhedged divergence loss.

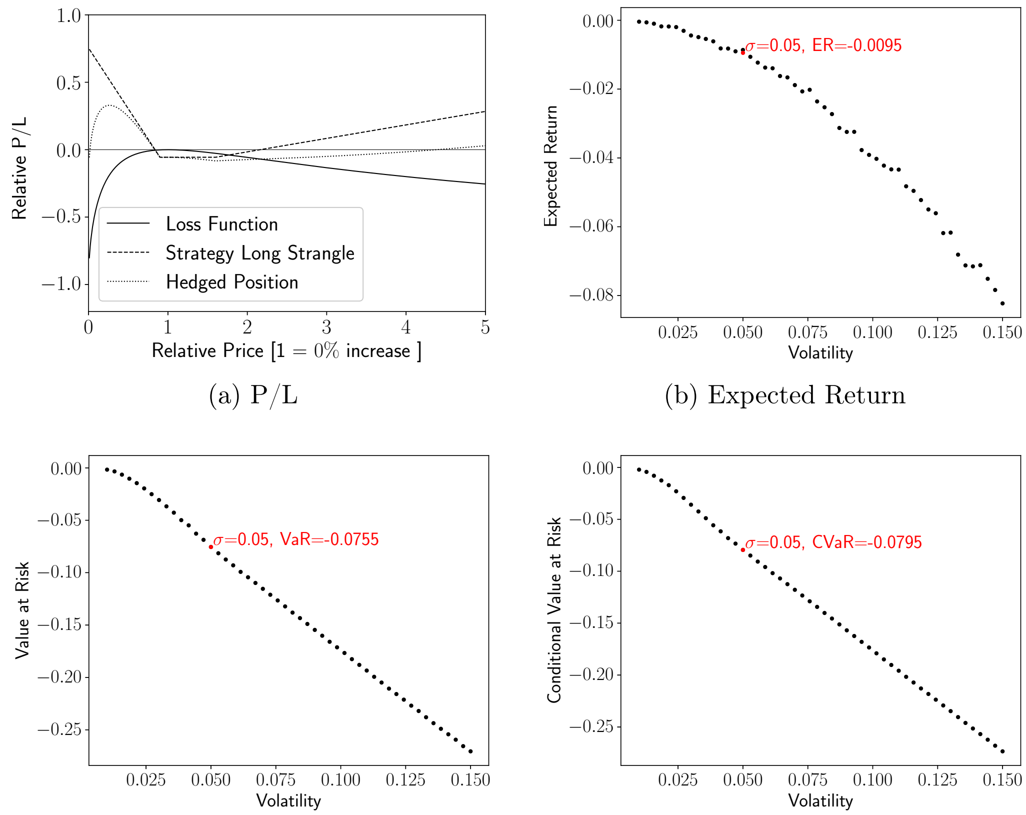
in a liquidity position. As seen in Fig. 3.1 the maximum loss is 100 % for price movements towards 0, and ∞ and the maximum profit is 0 for no price movement. This means this portfolio is constantly losing. The expected loss is -0.92%, the VaR is 3.49%, and the CVaR is 5.07%. We will now continue by analyzing the hedging strategies.

3.2.1 Strangle

The first strategy we will analyze is the simple strangle, which consists of one long call and one long put with $x_p \leq x_c$. When $x_p = x_c$ we call this a straddle. The strangle is interesting due to its simplicity compared to strategies we will analyze later. In addition, in low liquidity situations in the options market, thanks to only needing two strikes, it is easier to find the desired options.

We define this strategy as:

$$S_{\text{Strangle}} = w_p \cdot P(x_p) + w_c \cdot C(x_c)$$



(a) P/L (b) Expected Return (c) VaR (d) CVaR

Figure 3.2: Divergence Loss hedged with a Strangle with a Put with strike 0.9 and weight 0.9, and a call with strike 1.6 and weight 0.1.

where w_p and w_c are the weights of the two options and $P(x_p)$, $C(x_c)$ is a put with strike x_p or a call with strike x_c respectively.

As can be seen in Fig. 3.2 the hedged position seems to keep a flatter shape compared to the unhedged position except for the hump at $p_r < 1$. We expect such a hump since a linear approximation is never as good as a more complicated strategy.

The strategy has an expected loss of 0.95%, a VaR of 7.55%, and a CVaR of 7.95%. The maximum profit is 27% in the range $[0, 1]$ and ∞ in the range $[1, \infty]$ and the maximum loss is -15.5%.

Pricing Options

To price out options, we use standard Black-Scholes assumptions and the Black-Scholes formula[7]:

$$
\begin{aligned}
C(S,t) &= SN(d_1) - Ke^{-rt}N(d_2) \\
P(S,t) &= Ke^{-rt}N(-d_2) - SN(-d_1) \\
d_1 &= \frac{\ln(S/K) + (r + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} \\
d_2 &= d_1 - \sigma\sqrt{t}
\end{aligned}
$$

where C denotes the price of a call, P that of a put, S the price of the underlying at the time of purchase, K the strike price, r the risk-free rate, t the time to expiry, σ the annualized volatility of the underlying and N the standard normal cumulative probability function.

3.2.2 Power Perpetuals

This next strategy tries to improve on the strangle by following the curve more closely. We achieve a better approximation by applying the ideas of a Taylor approximation using derivatives; the equivalent derivatives to the monomials in the Taylor approximation are perpetual futures and power perpetuals.

Power Perpetuals We will briefly describe what a power perpetual is as mentioned in [8]. In short, a power perpetual tracks an index' returns and exponentiates it. That is, assume the underlying asset doubles in value, then the power perpetual that tracks the square will quadruple in value, whereas if the asset halves in value, the same power perpetual only loses a quarter of its value. We see the squared and cubed power perpetuals in Fig. 3.3

We can therefore see that on the domain $\mathcal{D} = \mathbb{R}^+$ the derivative behaves like an exponential function. With this in mind, we can quickly see that using a power perpetual to approximate a function via a Taylor approximation is doable. As a reminder, we define a Taylor approximation as:

$$f_n(x;a) = \sum_{i=0}^{n} \frac{f^{(i)}(a)}{i!}(x-a)^i + R_n(x;a) \tag{3.5}$$

where a is our approximation point, $f^{(i)}$ denotes the i-th derivative and $R_n(x;a)$ is the remainder function which is defined as $f(x) - f_n(x;a)$ and for $n \to \infty$ $R_n(x;a) = 0$.

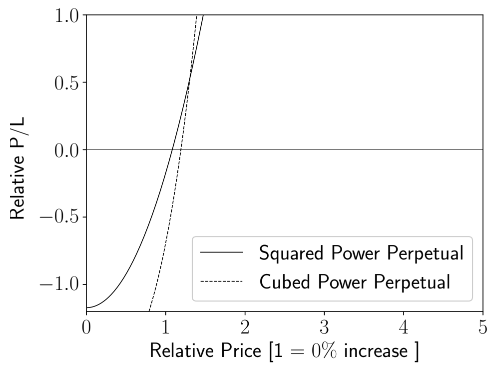


Figure 3.3: The P/L curve of a squared and cubed power perpetual.

Pricing Power perpetuals The power perpetuals are priced according to Black-Scholes assumptions as described in [7], the premium is then given by [9, 8]:

$$PP^n(S,t) = S^n \left(\frac{1}{2e^{-t\frac{n-1}{2}(2r+n\sigma^2)} - 1} - 1 \right)$$

where PP^n is the cost of the power perpetual of degree n over time t, S is the price of the underlying at time zero, and σ is the volatility of the underlying asset.

Construction To construct our approximation, we must identify the function's individual components. Let our function to approximate be the divergence loss function as shown in (Eq. 2.6). To be more precise, we will approximate $(f(x))^{-1}$, as to hedge the function, we are looking for the inverse. Below we will show the first two derivatives of the function:

$$f(x) = \frac{(1-\sqrt{x})^2}{1+x}, \qquad f(1) = 0$$

$$f^{(1)}(x) = \frac{x-1}{\sqrt{x}(x+1)^2}, \qquad f^{(1)}(1) = 0$$

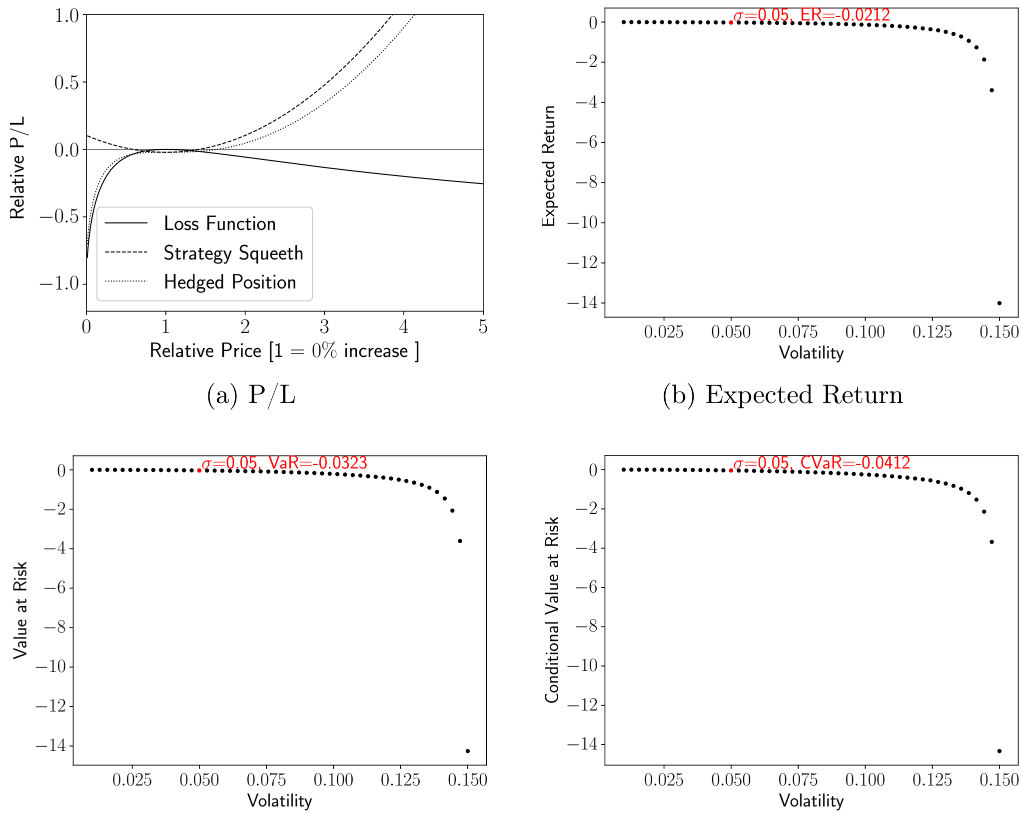
$$f^{(2)}(x) = -\frac{3x^2-6x-1}{2x^{3/2}(x+1)^3}, \qquad f^{(2)}(1) = \frac{1}{4}$$

Our approximation point will be at one since we want the best approximation to be close to the point of least change. Therefore we can see the derivatives evaluated at one above. When combining this with (Eq. 3.5), we get:

$$f_2(x;1) = 0 + 0 + \frac{1/4}{2}(x-1)^2$$

$$= \frac{1}{8}x^2 - \frac{1}{4}x + \frac{1}{8}$$
$$= \frac{1}{8}(x^2 - 1) - \frac{1}{4}(x - 1) \tag{3.6}$$

In Fig. 3.4 we can see that the approximation is excellent around one but quickly worsens towards the extremes. We could improve this by using a higher order power perpetual. However, the higher power comes with an increase in the cost of the strategy. More practically, no commercially available power perpetuals above the second power exist.



(a) P/L (b) Expected Return

(c) VaR (d) CVaR

Figure 3.4: Divergence loss hedged with a second order approximation using power perpetuals.

Strategy When constructing our strategy with a second order approximation, we start from Eq. 3.6, and as expected, we see a square and linear term. As mentioned above, we can approximate the square term using a squared power perpetual $(x^2 - 1)$ and the linear term using a perpetual future $(x - 1)$. We then construct our portfolio as follows, we go long one squared power perpetual with a weight of 0.125, and we go short a perpetual future with a weight of 0.25.

The portfolio is shown in Fig. 3.4. We see an expected loss of 2.12%, a VaR of 3.24%, and a CVaR of 4.12%. The maximum loss is −67.9%, and the maximum profit tends to infinity. During extreme volatility, the power perpetuals have very high premiums, and we see very low expected returns.

3.2.3 Series of Options

This strategy we will analyze has been proposed in [10, 11]. It consists in buying a series of puts and calls with infinitesimal strikes to perfectly approximate the function. We have slightly adapted the strategy to reflect the notation style in this thesis; however, the results remain the same. The adapted strategy can be represented as follows:

$$\begin{aligned} f(x;a) &= f(a)e^{-rT} + f^{(1)}(a)(x - ae^{-rT}) \\ &\quad + \int_0^a f^{(2)}(s)P(x;s)\ ds + \int_a^\infty f^{(2)}(s)C(x;s)\ ds \end{aligned} \tag{3.7}$$

As applied in Section 3.2.2 f is the divergence loss function, our approximation point a is one, and the risk-free rate r is 0. $P(x;s)$ and $C(x;s)$ define a put and a call respectively with strike s. Therefore, (Eq. 3.7) becomes:

$$\begin{aligned} f(x;1) &= 0 + 0 + \int_0^1 f^{(2)}(s)P(x;s)\ ds + \int_1^\infty f^{(2)}(s)C(x;s)\ ds \\ &= \int_0^1 -\frac{3s^2 - 6s - 1}{2s^{3/2}(s+1)^3}P(x;s)\ ds + \int_1^\infty -\frac{3s^2 - 6s - 1}{2s^{3/2}(s+1)^3}C(x;s)\ ds \end{aligned} \tag{3.8}$$

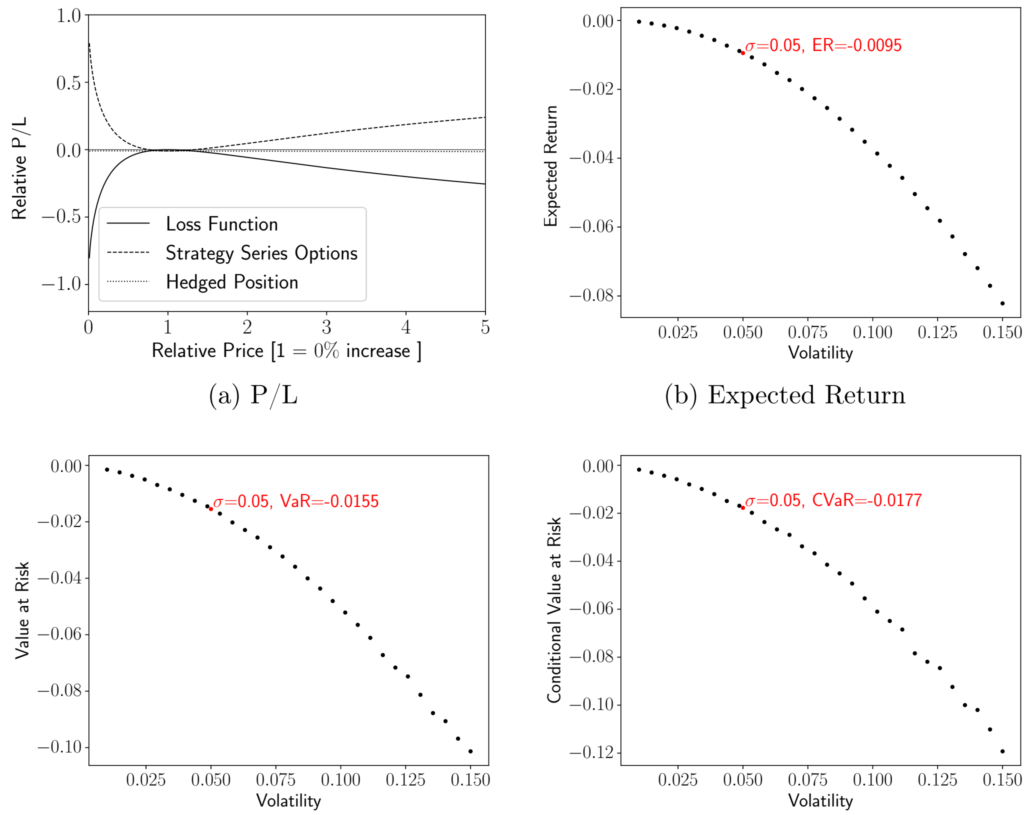
with the first two terms turning to 0 as we saw in Section 3.2.2. We can see Eq. 3.8 plotted in Fig. 3.5.

The strategy performs very well, with an expected loss of only −0.95%, a value at risk of 1.55% and CVaR of 1.77% with maximum profit and loss well within [0%, −2%] range in realistic outcomes.

3.3 Hedging the P/L of Liquidity Provision

To hedge the P/L of liquidity provisions, let us first see against what exactly we are hedging. As mentioned in Section 2.2.2, we are trying to hedge against the profit and losses of providing liquidity. It is defined by Eq. 2.7, and its profit and loss diagram can be seen in Fig. 3.6. We notice that the ideal area to hedge is in the interval between zero and one since we lose in this interval.

On average, we notice that this portfolio loses with an expected return of −0.85%. However, this strategy does come with a significant risk penalty; the value at risk



(a) P/L (b) Expected Return

(c) VaR (d) CVaR

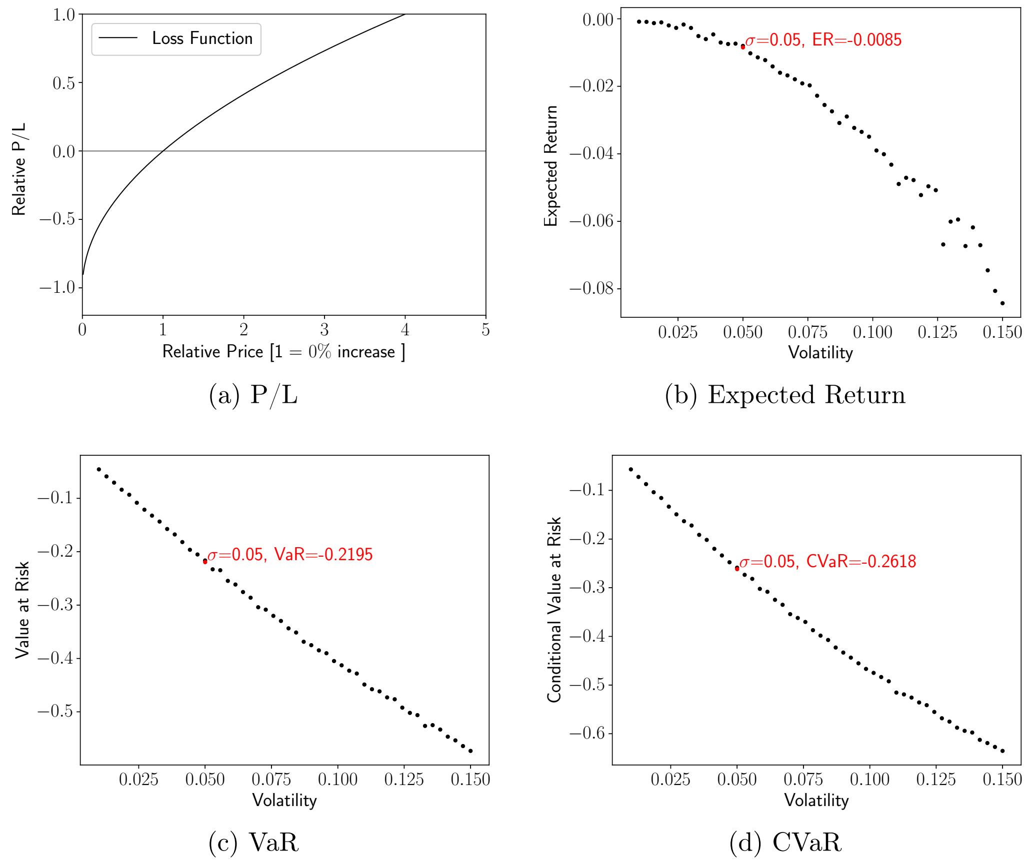
Figure 3.5: Strategy plotting a series of options as described in Eq. 3.8. The hedged position is very flat and hard to see.

is 21.95%, and the CVaR is 26.18%, which is not desirable. In addition, while the maximum profit is theoretically infinite, the maximum loss is 100%, which is bad. We, therefore, present three strategies to protect our portfolio, the first two are more straightforward strategies that should always be obtainable, and the other is a more complex strategy that is possibly not always obtainable.

3.3.1 Long Put

The first strategy consists of buying only a long put with the strike at 1. This strategy aims only to hedge the downside risk in the interval from zero to one while taking a hit on the upside profits. With the weight of the option, we can further define how much we are willing to lose on the downside. This downside is then min(weight, premium). The payoff diagram for weight one can be seen in Fig. 3.7.

The strategy graphed in Fig. 3.7 performs pretty well with an expected return of



(a) P/L (b) Expected Return

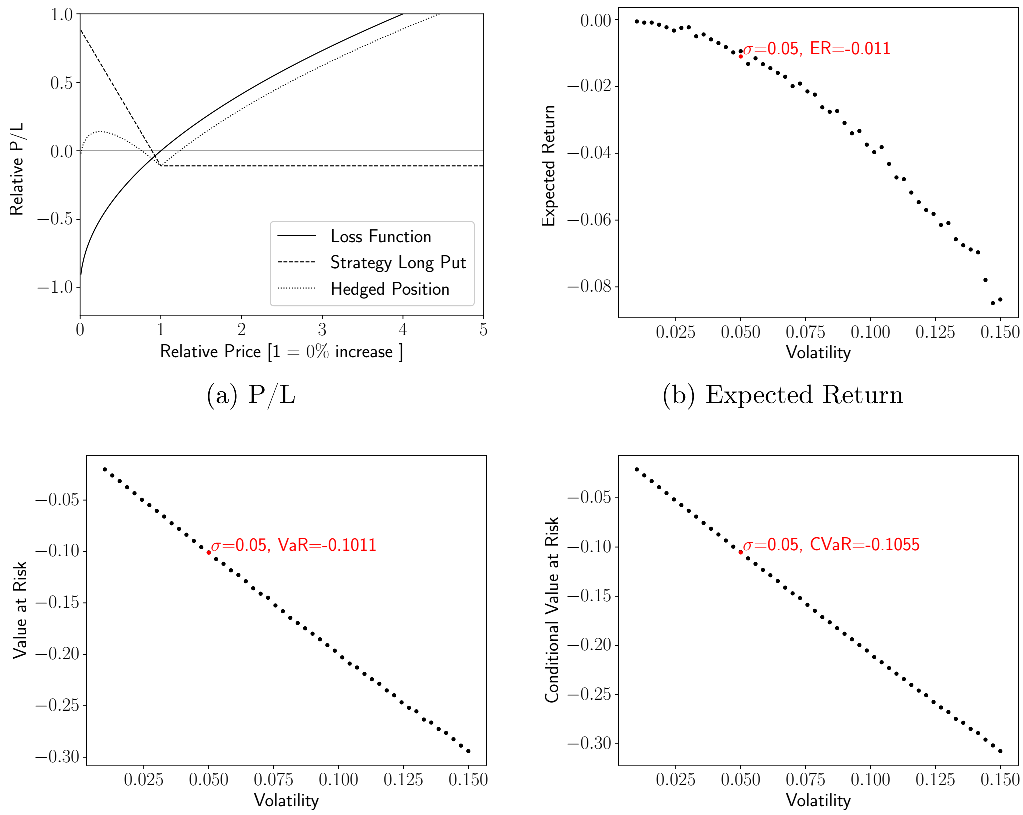
(c) VaR (d) CVaR

Figure 3.6: Performance metrics of the unhedged P/L of Liquidity Provision

−1.1%, a value at risk of 10.11% and a conditional value at risk of 10.55%, the strategy has a maximum loss right around 1 of 6.9% and a maximum profit that tends to infinity. We have significantly lowered our risk at the expense of some profits, which is what we wanted to achieve.

3.3.2 A different Power Perpetual Strategy

The second strategy we present is the equivalent of the strategy presented in Section 3.2.2. However, for the comparison against the P/L of liquidity provision, we can imagine this as the second-order approximation of the function given in Eq. 2.7. As noted in Section 3.2.2, we can compute the second order approximation by looking closer at the first and second derivative of Eq. 2.7. Our approximation point is at one. Therefore, if we combine all of this information, we get:



(a) P/L (b) Expected Return

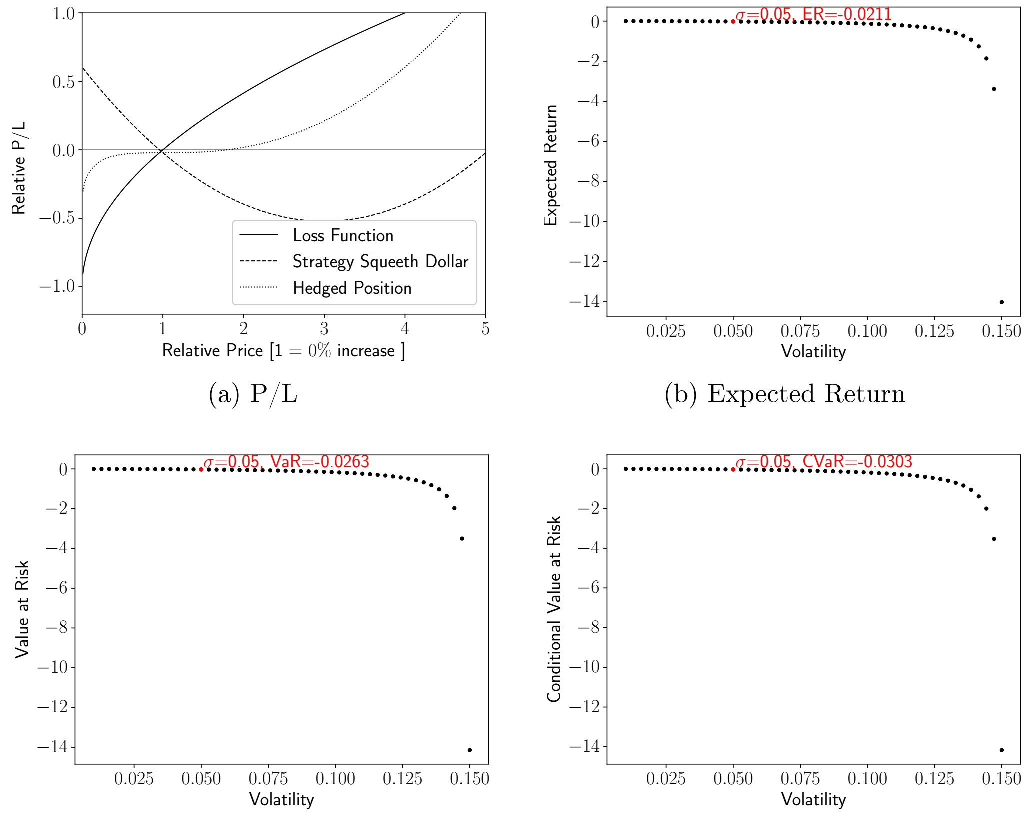
(c) VaR (d) CVaR

Figure 3.7: We see the dollar portfolio hedged with a long put with strike one and weight one. We also notice that the maximum downside is given by min(weight, premium).

$$\frac{d}{dx}\Delta\ell_D = -\frac{1}{2}\frac{1}{\sqrt{p_r}}$$
$$\frac{d^2}{dx}\Delta\ell_D = \frac{1}{4}\frac{1}{p_r^{3/2}}$$
$$f_1(x;1) = \frac{1}{8}(x^2-1) - \frac{3}{4}(x-1) \tag{3.9}$$

This strategy can be viewed in Fig. 3.8. We notice the linear term in Eq. 3.9, which tells us we should purchase a short forward with a weight of 0.75, and a squared term which implies we need to purchase a squared power perpetual with a weight of 0.125. We notice two points; first, the approximation looks very promising since it flattens around one but maintains the natural direction of the underlying curve. Second, this strategy only differs from the other power perpetual strategy by one short forward with a weight of 0.5.

Based on this analysis, we believe that any strategy defined for a liquidity provider's portfolio combined with a short forward with a weight of 0.5 will result in a strategy for a dollar portfolio with similar properties to its counterpart.



(a) P/L (b) Expected Return

(c) VaR (d) CVaR

Figure 3.8: The P/L of a liquidity provider hedged with a power perpetual of the second degree with a weight of 0.125 and a short forward of weight 0.75.

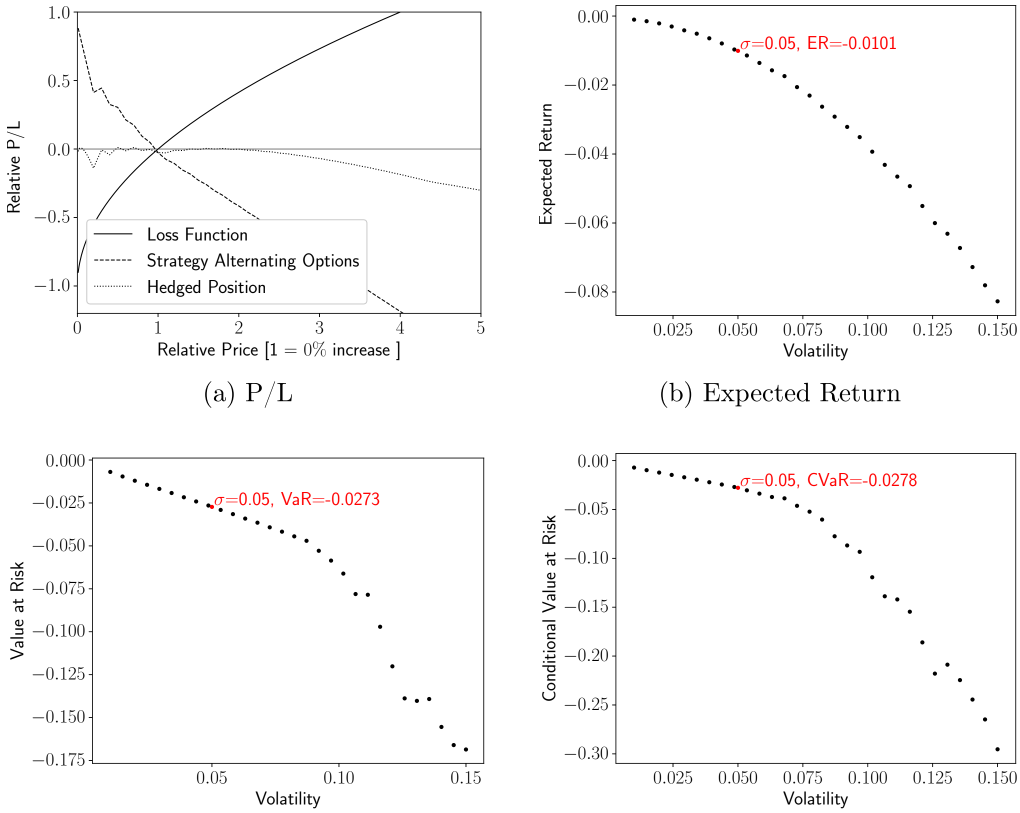
The performance of this strategy is even better. With an expected return of -2.11%, a VaR of 2.63% and a CVaR of 3.03%, the maximum profit tends to infinity, and the maximum loss stops at 64.39%

3.3.3 Alternating Options

Our last strategy consists of buying options by alternating short and long positions after every strike step. This will generate a step function that simulates the movement of the given loss function. We define this strategy as a sum of options as such:

$$f(x) = \frac{-1}{2}F(x) + \sum_{s \in \text{strikes}_P} w(s)P(x;s) + \sum_{s \in \text{strikes}_C} w(s)C(x;s) \tag{3.10}$$

where F defines a forward contract, $\text{strikes}_{P/C}$ represent the set of strikes being bought for the put and call respectively and $w(s) = \frac{1}{4}\frac{1}{s^{3/2}}$ alternates sign with every strike. In Fig. 3.9 we can see the above function with $\text{strikes}_P = \{0.1, 0.2, \ldots, 1\}$ and $\text{strikes}_C = \{1.1, 1.2, \ldots 9.9\}$.



(a) P/L (b) Expected Return

(c) VaR (d) CVaR

Figure 3.9: The P/L of a liquidity provider hedged with the alternating options strategy with $\text{strikes}_P = \{0.1, 0.2, \ldots, 1\}$ and $\text{strikes}_C = \{1.1, 1.2, \ldots 9.9\}$.

The strategy performs as desired and yields an expected return of -0.44% with a value at risk of 8.8%. The maximum profit is around 4.6% while the maximum loss tends to infinity.

3.4 Summary

We present all strategies as a table to visualize the differences between the strategies, highlighted in green in Tables 3.1 and 3.2 is the best performing strategy based on the goals set out.

Hedging Divergence Loss

Strategy	Expected Return	VaR	CVaR
Divergence Loss	-0.92%	3.49%	5.07%
Strangle	-0.95%	7.55%	7.95%
Power Perpetuals	-2.12%	3.23%	4.12%
Series of Options	-0.95%	1.55%	1.77%

Table 3.1: Summary of strategies hedging divergence loss.

Hedging P/L of Liquidity Provision

Strategy	Expected Return	VaR	CVaR
Liquidity Provision	-0.85%	21.95%	26.18%
Long Put	-1.1%	10.11%	10.55%
Power Perpetuals	-2.11%	2.63%	3.03%
Alternating Options	-1.01%	2.73%	2.78%

Table 3.2: Summary of strategies hedging the P/L of liquidity provision.

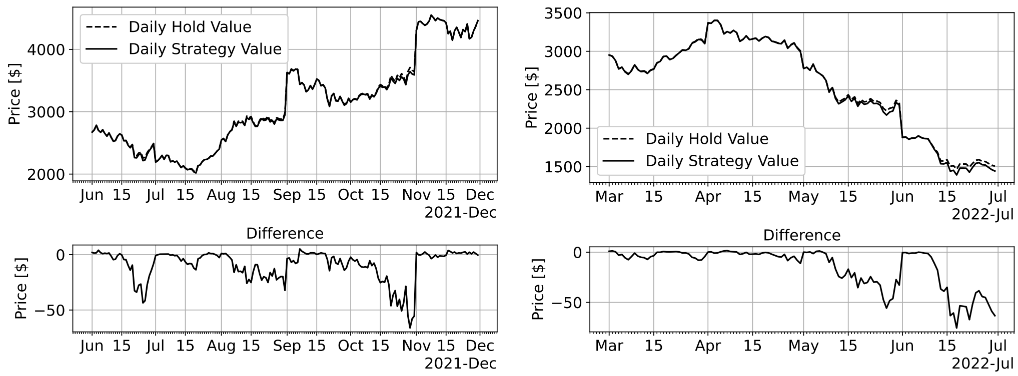
CHAPTER 4

Back-testing

All strategies were tested on data from 01-06-2021 to 30-11-2021 (referred to as first period or period one) and from 01-03-2022 to 30-06-2022 (referred to as second period or period two) on the ETH-USDC pool. Options were bought at the beginning of each month with an expiry on the last day of each month. Futures and squeeth investments are entered at the beginning of each month and liquidated at the end of each month, which means we never carry over positions to the following month. The assumption is that the initial investment into the pool is worth 1 ETH at the beginning of the testing period. The investor then provides liquidity for 0.5 ETH and the equivalent of 0.5 ETH in dollars. Strategies that do not employ options can be exited at any time; however, strategies that include options need to be held until the end of the month as the options do not allow for early expiry.

The value of the strategy always includes all associated costs and fees amassed from the strategy. We distribute costs paid in full at the beginning of the month (such as for options) over the whole month. Other costs and fees that are calculated daily are added to each day. The plotted data always measures end-of-day performance, which explains why the difference plot does not start at zero. The fees received for providing liquidity are approximated based on daily volume and total liquidity available in the market.

Our performance metrics are always based on the difference between the hold portfolio and strategy. In particular, our risk metrics are measured on the daily fluctuations of this difference, not on the difference itself. Profit and Loss (P/L) is measured at the end of each month, which means that while we might see daily fluctuations, the monthly result is what counts. Additionally, fees and costs are spread out throughout the month, as mentioned above; however, when calculating P/L, these are accumulated to the end of the month.



(a) Period from 01-06-2021 to 30-11-2021. (b) Period from 01-03-2022 to 30-06-2022.

Figure 4.1: The performance of unhedged divergence loss.

4.1 Hold Portfolio

First, we measured the performance of the unhedged divergence loss. We quickly notice in Figs. 4.1a and 4.1b that the strategy loses on most days, which is to be expected as the fees do not cover enough of the variability in such a portfolio. However, as we will see, the strategy is primarily profitable at the end of the month.

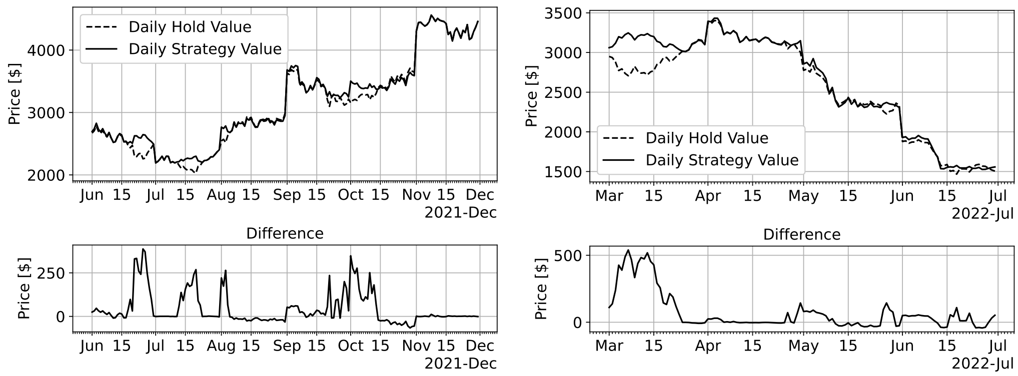
The strategy achieved a profit of $184.68 in period one and $12.72 in period two, so while the strategy loses throughout the month, it often comes back at the end of the month. The realized volatility is at 17% and 16% for the first and second periods, respectively, while the value at risk and conditional value at risk for both periods are very close, with value at risk at $41.56 and $55.77 and conditional value at risk of $50.71 and $63.51.

To beat the performance of the LP strategy, we want to achieve two key aspects. The first is to have a higher overall return, ideally always positive, and the second is to have a lower risk. We measured return as the absolute profit-and-loss and risk in terms of realized volatility (RV), value at risk, and conditional value at risk.

In the following subsections, we will test the strategies presented in Section 3.2.

4.1.1 Strangle

This strategy performed relatively poorly in the first period due to a disproportionately high cost for the strategy ($309.96 vs. $86.75 avg. for the month) during the first month of the test, losing $245.89 and losing $278.66 in total. This trend did not carry over in the second tested period (avg. cost down to $33.05), during which the strategy netted $162.06 in profits (up from $12.72 in the hold



(a) Period from 01-06-2021 to 30-11-2021. (b) Period from 01-03-2022 to 30-06-2022.

Figure 4.2: Daily movements of the hold portfolio, plotted against the strangle strategy's daily value and the difference between the two.

strategy), which could imply this is an anomaly.

Divergence loss was virtually eliminated as can be seen in Figs. 4.2a and 4.2b. There were only short periods in which the strategy performed negatively. These were primarily periods with low volatility, where the costs were high. However, since the strategy predicted positive returns on high volatility, we primarily see periods of positive returns.

The strategy yielded reductions in the value at risk in both periods, \$36.63 (vs. \$41.56) and \$35.00 (vs. \$55.77), as well as a reduction in the CVaR for both periods, \$49.94 (vs. \$50.71) and \$39.31 (vs. \$64.51). We also notice a slight reduction in volatility, sinking to 17% and 14%, respectively.

Compared to the liquidity provider's strategy, this strategy brought some of the desired results to the table. The strategy has delivered lower risks, however, it also brought a disproportional reduction in profits, which is not desired.

4.1.2 Squeeth (ETH Power Perpetual)

As the backtesting was performed on the ETH-USDC pool, we used squeeth in our strategy, which is an implementation of a squared power perpetual tracking ETH. Squeeth requires daily funding payments to counter-parties which are automatically removed from one's position. Additionally, since this derivative has only existed since January 2022, this strategy has only been tested from 01-03-2022 to 30-06-2022.

The squeeth strategy has some days when it performs negatively, as seen in Fig. 4.3. However, the strategy mainly performed positively. Due to the convex nature of squeeth, we notice that the performance grows quickly with small upward movements and falls slowly with downward movements. The strategy

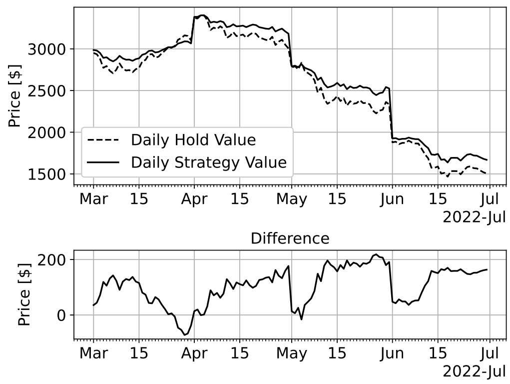


Figure 4.3: The performance of the squeeth strategy from 01-03-2022 to 30-06-2022.

yielded \$591.45 (vs. \$12.72 for hold) in the tested period, with a volatility of 15% (vs. 16%). With this strategy came a significant reduction in the VaR to \$5.95 (vs. \$55.77) and a slight reduction in CVaR to \$42.72 (vs. \$63.51).

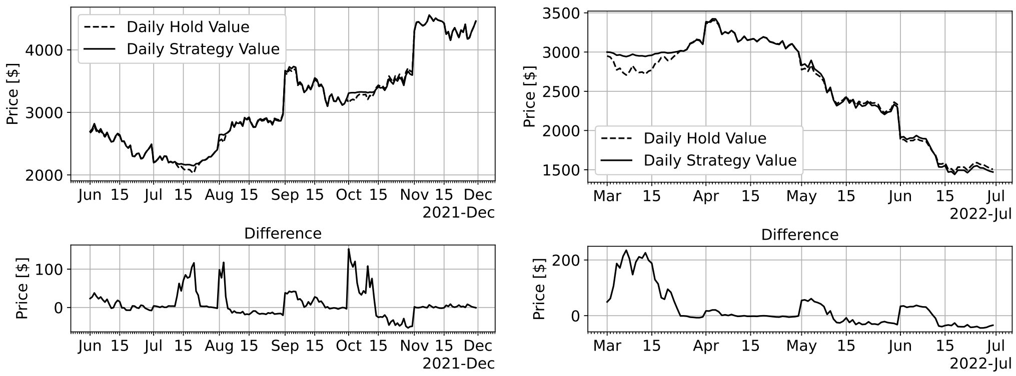
The squeeth strategy seems to have fared very well in the tested period. The profits are up, while the risk is down. However, due to the limited testing period, we cannot conclusively say that this strategy will always perform well.

4.1.3 Sum of Options

The strategy presented in Section 3.2.3 requires an infinite amount of options. Since we cannot realistically replicate such a strategy, we adapted the strategy to be a sum of options at specific intervals. We decided to apply the strategy to options in the range of −50% to +100% in intervals of 10% of the starting price of Ethereum.

This new strategy performed as expected, maintaining a flat profile on most days as can be seen in Figs. 4.4a and 4.4b. However, this also means there was a reduction in profits. The strategy netted a profit of \$143.01 (vs. \$184.68) in the first period and \$27.71 (vs. \$12.72) in the second period while reducing the VaR to \$36.82 and \$39.68, respectively. We see a similar trend in the CVaR with a reduction to \$46.20 in period one and \$41.72 in period two. The real benefit of this strategy is its cost, averaging \$9.04 per month.

We can conclude that the strategy delivered on the expected goals in the tested periods. The profits remain within a reasonable distance of the unhedged position, while the risk metrics are lower in comparison. In addition, the strategy's costs remain low and do not have sudden spikes as the strangle strategy had.



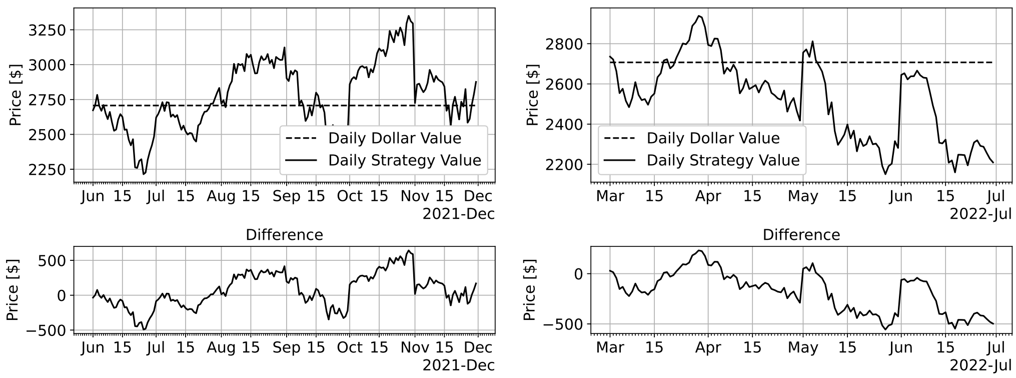
(a) Period from 01-06-2021 to 30-11-2021. (b) Period from 01-03-2022 to 30-06-2022.

Figure 4.4: The performance of the adapted series of options strategy.

4.2 Liquidity Provider Portfolio

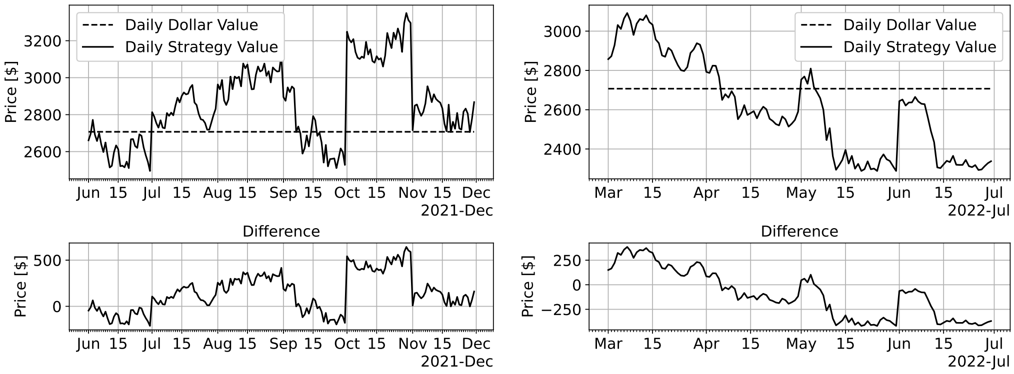
To compare all the other strategies, we will first look at the performance of the dollar portfolio in the tested periods. First, note that the portfolio's value remains the same throughout each month, meaning no money is invested in the dollar portfolio. We can see this behavior in Fig. 4.5. In particular, that means that by investing, we move from not being exposed to any market movements to exposure to the markets. Our goal is then to minimize this market exposure.

In total, the strategy resulted in \$1109.49 in profit in the first period and \$902.48 in losses in the second period. We saw average monthly volatility of 17% and 15% in the first and second periods, respectively. The VaR is at \$327.62 and \$499.05, respectively, and the CVaR of the portfolio stands at \$404.13 for the first period and \$521.59 for the second period.



(a) Period from 01-06-2021 to 30-11-2021. (b) Period from 01-03-2022 to 30-06-2022.

Figure 4.5: The performance of the dollar portfolio.



(a) Period from 01-06-2021 to 30-11-2021. (b) Period from 01-03-2022 to 30-06-2022.

Figure 4.6: The performance of the long put strategy with a strike close to the starting price.

4.2.1 Long Put

For this strategy, our target was to find an at-the-money put. We notice that we are still exposed to similar fluctuations as in the unhedged portfolio. This behavior is expected as the put only protects our downside risk by lowering our upside potential. We notice this in Figs. 4.6a and 4.6b compared to Figs. 4.5a and 4.5b where our upside and downside is typically halved.

This strategy yields profits of $317.97 in the first period while losing $734.86 in the second. As expected our profits are down, however so are also our losses. In that regard, our volatility sank for the second period to 12% (vs. 15% in the dollar portfolio) while remaining the same in the first period. The VaR sank as well, moving down to $179.36 (vs. $327.62) in the first period and to $410.26 (vs. $499.05) in the second period.

Buying the put seems to have had the desired effect in the tested periods, the losses are reduced at the expense of some profits, and we have also reduced the risk metrics. The simplicity of this strategy comes at a cost in terms of a high premium on the option.

4.2.2 Squeeth Strategy versus Dollar Portfolio

This strategy is, as described in Section 4.1.2, using the ETH^2 power perpetual squeeth. We immediately notice that the performance is practically but not completely identical to the squeeth strategy presented in Section 4.1.2. As explained in Section 3.3.2, buying a short forward with a weight of 0.5, in addition to the dollar strategy, effectively simulates the hold strategy up to a small additive constant.

The profits of this strategy compared to both the dollar portfolio and the squeeth strategy used in the hold portfolio is higher at \$614.72 as compared to \$-902.48 and \$591.45 for the two other strategies, respectively. In addition, we see lower volatility than both other strategies at 5% (vs. 15% for the others). The value at risk is significantly lower than the unhedged dollar portfolio, at \$5.95, and the conditional value at risk is down to \$42.72. These values are identical to the squeeth strategy used to hedge against the hold portfolio.

The analysis of this strategy is the same as for the other squeeth strategy. However, the strategy is much more recommendable in this situation due to the significantly lower risk metrics compared to the unhedged LP portfolio.

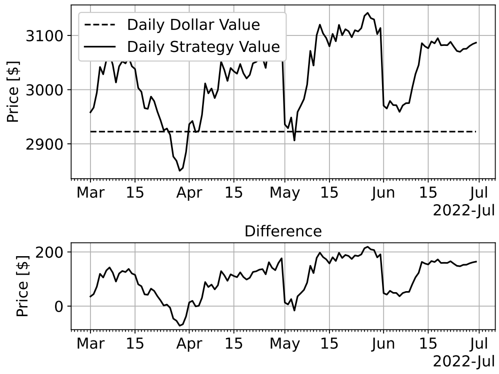
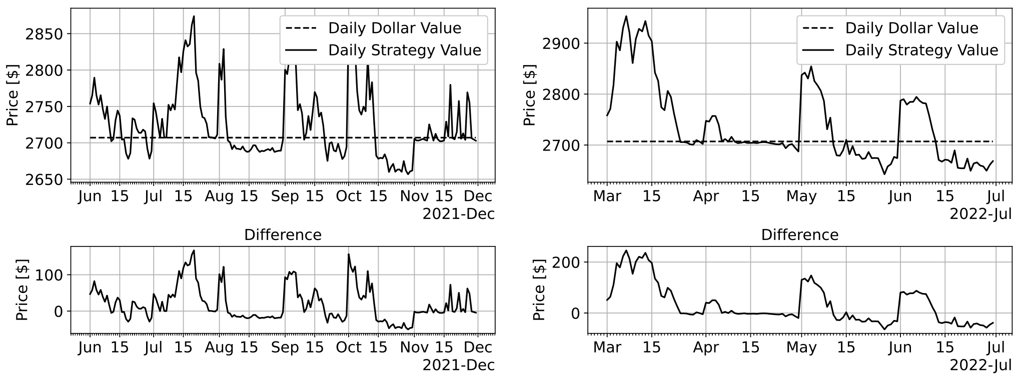


Figure 4.7: Performance of the squeeth strategy vs. a dollar portfolio during the period from 01-03-2022 to 30-06-2022.

4.2.3 Alternating Options



(a) Period from 01-06-2021 to 30-11-2021. (b) Period from 01-03-2022 to 30-06-2022.

Figure 4.8: The performance of the alternating options strategy as compared to the dollar portfolio.

The alternating options strategy was executed as described in Section 3.3.3, the only difference being that we limited our search for options in the range of strikes of [-50%, +100%]. The strategy performed relatively poorly, lowering profits and losses significantly compared to the unhedged variant, returning $-446.33 (vs. $1109.49) in the first period and $-168.65 (vs. $-902.48) in the second period. The VaR and CVaR were down drastically compared to the unhedged strategy at $40.24 and $49.08, respectively, for the VaR and $45.58 and $55.14, respectively for the CVaR.

This strategy did not achieve the goal of this thesis as the risk is lower compared to the unhedged portfolio while hedging away all profits, this means that we should rather invest unhedged instead of using this strategy. In Chapter 5 we will present further details into why we believe some of these strategies did not perform similarly to their theoretical counterparts.

Chapter 5

Results

We looked at several strategies in the previous two chapters and analyzed their performance. Now that we have seen all these strategies individually, we would like to leave some final words on the strategies as a whole. In the first section, we will discuss our test results and try to elect a winner for possible scenarios. In the second part, we will explain some key obstacles and limitations we noticed during testing that might have affected the results.

5.1 Analysis and Recommendations

In general, we noticed that the two unhedged strategies performed reasonably well on their own. However, some strategies outperformed the two unhedged strategies in terms of risk. In the theoretical part, we saw that the two strategies using more than two options were the best in reducing risk at the expense of some profits. This did not ultimately carry over into the empirical tests as both strategies seemed to struggle to match the theoretical strategies to actually available assets.

The strategies employing power perpetuals fared very well in both the theoretical part, coming in second place, and the empirical tests. The risk is lower, while the profits are not reduced significantly.

We also saw that the more straightforward strategies fared decently well but could not hold their weight against the more complicated strategies.

For both scenarios using squeeth seems to be a very viable solution; it is very flexible, can be exited at any time of the month, and seems to provide the right amount of hedging. In comparison, strategies employing options are less recommendable, leading us to the next section.

5.2 Obstacles and Limitations

During testing, we noticed several limitations to strategies employing options. While options are typically a widely available derivative in traditional finance, they are losing trend in decentralized finance. During our testing, we noticed several periods in which the availability of options at given strikes was very limited. This severely lowered the performance of several strategies using options. Additionally, as noted in the strangle strategy, we had some months in which strategy costs were disproportionately high. This is presumably due to the low volume of options at specific strikes causing higher prices. While we would like to test the optimal real-life performance of these strategies, we noticed that the pricing did not often reflect Black-Scholes pricing and hence would not enable a fair comparison to the presented strategies.

In addition, squeeth is a relatively new derivative, launched at the beginning of 2022. Therefore, while the results seem promising in the tested period, it is not enough data to conclusively say whether this strategy is as good as it seems in the tests. In theory, we expect the strategy to perform just as well as it did in the tested period. However, the theory does not always imply practice.

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Appendix A

Computing Divergence Loss

A.1 Divergence Loss

Divergence loss is the difference in the value of holding the assets compared to providing liquidity. Therefore, we first set out to define these two quantities. Let V_0 be the initial value of both the positions and let $V_H(t)$ denote the value of holding at time t and $V_{LP}(t)$ the value of the liquidity position at time t.

In addition, we assume p_0 to be the price of Asset A in terms of Asset B at time 0, p_t at time t. Let $a_0,\ b_0$ be the starting amount of Asset A and Asset B respectively, and $a_t,\ b_t$ at time t.

$$\begin{aligned} V_0 &= p_0 \cdot a_0 + b_0 \\ V_H(t) &= p_t \cdot a_0 + b_0 && \text{(A.1)} \\ V_{LP}(t) &= p_t \cdot a_t + b_t && \text{(A.2)} \\ \ell_{LP} &= V_H(t) - V_{LP}(t) && \text{(A.3)} \end{aligned}$$

We define, without loss of generality, that $a_t = a_0 + \Delta a$ and $b_t = b_0 - \Delta b$, we can rewrite b_t as

$$\begin{aligned} b_t &= b_0 - \Delta b \\ &= b_0 - b_0 \frac{\Delta a}{a + \Delta a} \\ &= b_0 \frac{a_0}{a_0 + \Delta a} && \text{(A.4)} \\ \Delta b &= b_0 \frac{\Delta a}{a_0 + \Delta a} && \text{(A.5)} \end{aligned}$$

As discussed in Section 2.1.1, we know p_0 and p_t hence we can compute

$$\begin{aligned} \ell_{LP} &= V_H(t) - V_{LP}(t) \\ &= p_t \cdot a_0 + b_0 - (p_t \cdot a_t + b_t) && \text{((A.1) and (A.2))} \end{aligned}$$

$$
\begin{aligned}
&= p_t \cdot a_0 - p_t \cdot a_t + b_0 - b_t \\
&= \frac{b_t}{a_t} a_0 - \frac{b_t}{a_t} a_t + b_0 - b_t \\
&= b_t \frac{a_0}{a_t} + b_0 - 2b_t \\
&= (b_0 - \Delta b) \frac{a_0}{a_t} - 2(b_0 - \Delta b) + b_0 \\
&= b_0 \left(\frac{a_0}{a_t} - 1 \right) + \Delta b \left(2 - \frac{a_0}{a_t} \right) \\
&= b_0 \left(\frac{a_0}{a_0 + \Delta a} - 1 \right) + b_0 \frac{\Delta a}{a_0 + \Delta a} \left(2 - \frac{a_0}{a_0 + \Delta a} \right) \\
&= b_0 \left(\frac{2\Delta a}{a_0 + \Delta a} - \frac{\Delta a}{a_0 + \Delta a} - \frac{\Delta a a_0}{(a_0 + \Delta a)^2} \right) \\
&= b_0 \left(\frac{2\Delta a a_0 + 2\Delta a^2 - \Delta a a_0 - \Delta a^2 - \Delta a a_0}{(a_0 + \Delta a)^2} \right) \\
&= b_0 \left(\frac{\Delta a}{a_0 + \Delta a} \right)^2 = b_0 \left(1 - \frac{a_0}{a_0 + \Delta a} \right)^2
\end{aligned}
$$

Next, we introduce the price ratio at time t to the price at time 1, defined as p_r.

$$
\begin{aligned}
p_r = \frac{p_t}{p_0} &= \frac{b_t}{a_t} \frac{a_0}{b_0} = \frac{b_0 - \Delta b}{a_0 + \Delta a} \frac{a_0}{b_0} \\
&= \frac{b_0 a_0 \left(1 - \frac{\Delta a}{a_0 + \Delta a} \right)}{b_0 (a_0 + \Delta a)} = \frac{a_0 \frac{a_0}{a_0 + \Delta a}}{a_0 + \Delta a} \\
&= \frac{a_0^2}{(a_0 + \Delta a)^2} = \left(\frac{a_0}{a_0 + \Delta a} \right)^2
\end{aligned} \tag{A.6}
$$

Lastly, we want to compute the relative divergence loss. We achieve this by dividing the result above by the value of the hold position.

$$
\begin{aligned}
\Delta \ell_{LP} = \frac{\ell_{LP}}{V_H(t)} &= \frac{b_0 \left(1 - \frac{a_0}{a_0 + \Delta a} \right)^2}{p_t \cdot a_0 + b_0} = \frac{b_0 \left(1 - \sqrt{p_r} \right)^2}{\frac{b_0 - \Delta b}{a_0 + \Delta a} a_0 + b_0} \\
&= \frac{b_0 \left(1 - \sqrt{p_r} \right)^2}{\frac{a_0 b_0 - a_0 b_0 \frac{\Delta a}{a_0 + \Delta a}}{a_0 + \Delta a} + b_0} = \frac{b_0 \left(1 - \sqrt{p_r} \right)^2}{b_0 \frac{\frac{a_0^2 + a_0 \Delta a - a_0 \Delta a}{a_0 + \Delta a}}{a_0 + \Delta a} + b_0} \\
&= \frac{\left(1 - \sqrt{p_r} \right)^2}{\frac{a_0^2}{(a_0 + \Delta a)^2} + 1}
\end{aligned}
$$

$$= \frac{(1-\sqrt{p_r})^2}{1+p_r} \tag{A.7}$$

Since we want our loss function to output negative numbers, we multiply Eq. A.7 by negative one to obtain:

$$-\Delta\ell_{LP} = -\frac{(1-\sqrt{p_r})^2}{1+p_r} = \frac{2\sqrt{p_r}-1-p_r}{1+p_r} = \frac{2\sqrt{p_r}}{1+p_r} - 1$$

A.2 Profit and Loss of Liquidity Position

Compared to the previous calculations, we compare the initial value of the liquidity position. Hence $V_D(t) = V_0$. Performing the same calculations as above, we obtain:

$$\begin{aligned}
\ell_D &= V_D(t) - V_{LP}(t) = (p_0 \cdot a_0 + b_0) - (p_t \cdot a_t + b_t) \\
&= p_0 \cdot a_0 + b_0 - \frac{b_t}{a_t} a_t - b_t = p_0 \cdot a_0 + b_0 - 2b_t \\
&= \frac{b_0}{a_0} a_0 + b_0 - 2(b_0 - \Delta b) = 2b_0 - 2b_0 + 2\Delta b \\
&= 2\Delta b = 2b_0 \frac{\Delta a}{a_0 + \Delta a} = 2b_0 \left(1 - \frac{a_0}{a_0 + \Delta a}\right) = 2b_0(1 - \sqrt{p_r})
\end{aligned}$$

Next, as before, we want to compute the relative loss, which means we divide our result above by $V_D(t)$

$$\begin{aligned}
\Delta\ell_D &= \frac{2b_0(1-\sqrt{p_r})}{V_D(t)} = \frac{2b_0(1-\sqrt{p_r})}{p_0 \cdot a_0 + b_0} = \frac{2b_0(1-\sqrt{p_r})}{\frac{b_0}{a_0} a_0 + b_0} = \frac{2b_0(1-\sqrt{p_r})}{2b_0} \\
&= 1 - \sqrt{p_r}
\end{aligned}$$

As before, we are interested in the negative loss; hence we multiply the above result by negative one and obtain:

$$-\Delta\ell_D = -(1-\sqrt{p_r}) = \sqrt{p_r} - 1$$

Appendix B

Data Back-Testing

Here we present the data from the back-testing, RVar is the realized variance in percentage and RV is the realized volatility in percentage. Columns marked with (Total) signify that instead of an average at the end it displays the total.

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2021-06	0.02	0.15	-41.56	-42.58	0.00
2021-07	0.03	0.17	-12.15	-12.93	0.00
2021-08	0.02	0.13	-25.95	-29.12	0.00
2021-09	0.06	0.25	-14.14	-14.34	0.00
2021-10	0.01	0.09	-57.91	-62.05	0.00
2021-11	0.05	0.22	-2.35	-3.00	0.00
Average	0.03	0.17	-41.56	-50.71	0.00

(a) Period from 01-06-2021 to 30-11-2021

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2022-03	0.01	0.09	-7.74	-7.89	0.00
2022-04	0.01	0.12	-8.41	-9.78	0.00
2022-05	0.03	0.17	-48.23	-52.00	0.00
2022-06	0.07	0.26	-67.22	-71.39	0.00
Average	0.03	0.16	-55.77	-63.51	0.00

(b) Period from 01-03-2022 to 30-06-2022

Table B.1: Data for the unhedged hold portfolio.

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2021-06	0.01	0.12	-8.41	-9.12	309.96
2021-07	0.02	0.16	-1.01	-1.86	1.14
2021-08	0.03	0.18	-25.99	-29.16	1.27
2021-09	0.07	0.26	-9.38	-9.75	191.63
2021-10	0.01	0.12	-57.97	-62.11	1.65
2021-11	0.05	0.22	-2.19	-2.26	23.38
Average	0.03	0.17	-36.63	-49.94	86.75

(a) Period from 01-06-2021 to 30-11-2021

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2022-03	0.00	0.06	-7.79	-7.94	1.46
2022-04	0.01	0.12	-5.45	-5.64	29.71
2022-05	0.03	0.17	-32.51	-33.04	70.08
2022-06	0.05	0.22	-41.56	-42.15	30.75
Average	0.02	0.14	-36.00	-39.31	33.05

(b) Period from 01-03-2022 to 30-06-2022

Table B.2: Data for the strangle hedge.

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2022-03	0.00	0.04	-66.91	-69.57	6.03
2022-04	0.01	0.11	1.36	0.33	4.37
2022-05	0.02	0.15	6.30	-5.04	8.67
2022-06	0.08	0.29	42.37	39.27	8.35
Average	0.03	0.15	-5.95	-42.72	6.86

Table B.3: Period from 01-03-2022 to 30-06-2022

Table B.4: Data for the squeeth hedge in the period from 01-03-2022 to 30-06-2022

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2021-06	0.02	0.14	-7.13	-7.52	30.31
2021-07	0.02	0.15	-0.83	-1.47	1.70
2021-08	0.02	0.15	-18.52	-19.41	3.80
2021-09	0.07	0.26	-3.50	-3.75	14.10
2021-10	0.01	0.09	-49.72	-51.49	7.28
2021-11	0.05	0.21	-1.67	-1.74	8.18
Average	0.03	0.16	-36.82	-46.20	10.79

(a) Period from 01-06-2021 to 30-11-2021

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2022-03	0.00	0.04	-7.11	-7.11	1.33
2022-04	0.01	0.12	-4.89	-4.96	2.60
2022-05	0.03	0.16	-32.12	-32.15	11.09
2022-06	0.06	0.24	-43.89	-43.99	10.63
Average	0.02	0.14	-39.68	-41.72	6.41

(b) Period from 01-03-2022 to 30-06-2022

Table B.5: Data for the series of options hedge.

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2021-06	0.02	0.15	-480.36	-486.45	0.00
2021-07	0.01	0.11	-241.01	-249.81	0.00
2021-08	0.01	0.12	12.22	0.05	0.00
2021-09	0.04	0.19	-327.62	-337.93	0.00
2021-10	0.03	0.17	188.66	170.78	0.00
2021-11	0.07	0.26	-123.20	-134.56	0.00
Average	0.03	0.17	-327.62	-404.13	0.00

(a) Period from 01-06-2021 to 30-11-2021

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2022-03	0.01	0.10	-210.88	-216.96	0.00
2022-04	0.01	0.11	-245.16	-267.37	0.00
2022-05	0.04	0.20	-516.33	-536.41	0.00
2022-06	0.03	0.19	-512.47	-529.92	0.00
Average	0.02	0.15	-499.05	-521.59	0.00

(b) Period from 01-03-2022 to 30-06-2022

Table B.6: Data for the unhedged dollar portfolio.

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2021-06	0.01	0.12	-196.48	-204.21	343.80
2021-07	0.02	0.14	11.67	11.55	47.81
2021-08	0.01	0.12	169.40	157.25	1.27
2021-09	0.03	0.17	-186.91	-191.65	212.73
2021-10	0.07	0.27	374.10	363.52	1.50
2021-11	0.06	0.24	-1.14	-1.25	257.40
Average	0.03	0.17	-179.36	-190.97	142.00

(a) Period from 01-06-2021 to 30-11-2021

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2022-03	0.01	0.08	94.16	92.06	1.46
2022-04	0.01	0.09	-186.82	-189.97	32.83
2022-05	0.02	0.14	-418.87	-418.90	77.71
2022-06	0.03	0.17	-411.50	-412.69	32.99
Average	0.02	0.12	-410.26	-415.05	36.30

(b) Period from 01-03-2022 to 30-06-2022

Table B.7: Data for the long put hedge.

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2022-03	0.00	0.04	-66.91	-69.57	6.03
2022-04	0.00	0.04	1.22	0.20	4.37
2022-05	0.00	0.07	6.33	-5.00	8.67
2022-06	0.00	0.05	42.60	39.51	8.35
Average	0.00	0.05	-5.95	-42.72	6.86

Table B.8: Period from 01-03-2022 to 30-06-2022

Table B.9: Data for the squeeth hedge of the dollar portfolio in the period from 01-03-2022 to 30-06-2022

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2021-06	0.00	0.04	-28.90	-28.98	266.13
2021-07	0.00	0.05	-0.23	-0.69	17.07
2021-08	0.00	0.05	-19.61	-19.62	46.81
2021-09	0.00	0.05	-29.46	-30.72	129.28
2021-10	0.01	0.08	-47.21	-48.78	102.26
2021-11	0.00	0.06	-4.90	-4.92	80.90
Average	0.00	0.06	-40.24	-45.58	106.23

(a) Period from 01-06-2021 to 30-11-2021

Date	RVar	RV	VaR (Total)	CVaR (Total)	Cost
2022-03	0.00	0.06	-5.59	-6.02	14.70
2022-04	0.00	0.02	-13.27	-16.52	24.72
2022-05	0.00	0.07	-49.08	-56.78	90.17
2022-06	0.00	0.05	-57.52	-57.57	85.47
Average	0.00	0.05	-49.08	-55.14	53.74

(b) Period from 01-03-2022 to 30-06-2022

Table B.10: Data for the alternating options hedge.